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XXII. *Introduction to the First Guthrie Lecture.* *By Prof.* G. Carey-Foster, F.R.S.

I have been asked, as one of the oldest members of this Society, to say a few words, by way of preface to the first "Guthrie Lecture," in order to give some idea of Frederick Guthrie's place as a scientific man and why it is appropriate that his name should be associated with the series of lectures which the Council have decided to institute.

Guthrie was born in London in 1833 and died in London in 1886. Like many men who have attained distinction in physics, as Faraday, Regnault and the present President of the Royal Society, Guthrie began his scientific career as a chemist. After a general course at University College, London, and taking his B.A. degree in 1852, he went to Heidelberg to work at chemistry under Bunsen and then to Marburg to work under Kolbe. Here he took the degree of Ph.D. After returning to England he acted as assistant, first to Frankland at Manchester, at the then recently founded Owens College, then to Lyon Playfair at Edinburgh. His first independent appointment came in 1861, when he became Professor of Physics and Chemistry at the Royal College, Mauritius.

In 1869 he received the appointment which he retained for the rest of his life, that of the Professorship of Physics in the Royal School of Mines, the institution which, eventually, gave rise to the great Imperial College of Science and Technology.

Two generations ago, when Guthrie was at the beginning of his career, physics was a much more purely experimental and descriptive science than it is now. The greater part of modern mathematical physics did not exist. It is true that Laplace, Fresnel, Fourier and Ampère had published their marvellous works, but, in order to appreciate the change that has taken place since that time, it is sufficient to call to mind that the idea of the Conservation of Energy was then only making its way and was waiting for its necessary foundation, a system of absolute measurement for thermal, magnetic and electric phenomena to be firmly laid. The first momentous step towards such a system had been taken by Gauss in the case of magnetism and was being extended by Wilhelm Weber to electricity. Joule's work was still in progress. Thomson, Clausius and Rankine were laying the foundations of thermodynamics and of the application of mathematical methods to electricity. Green's work was being re-discovered. But all these things were still matters for discussion among the leaders;

they were not yet included in the settled scientific territory to which a student had access.

The difference between the state of physics in Guthrie's early time and in the present becomes again strikingly apparent when we remember that Thomson and Tait's "Natural Philosophy" and Maxwell's "Electricity" were neither of them published till a good many years later. In those days, the best treatises (in English, at least) on Heat, Light and Electricity were the introductory parts of some of the larger books on Chemistry (*e.g.*, Vol. I. of Miller's "Chemistry"). In those days too, a chemical laboratory was the only school of scientific manipulation.

I have been led to this rather long digression by remembering that the transformation that Guthrie underwent, from chemist into physicist, was a fairly common phenomenon among his predecessors and contemporaries.

All Guthrie's work was characterised by independence and originality, and he had a remarkable gift for recognising that there was something worth studying even in the most familiar occurrences. He is probably best known by the results embodied in the series of Papers, published in the "Proceedings" of this Society, on "Salt Solutions and Attached Water," which led him to the discovery of the Cryo-hydrates and Eutectic Alloys. His experiments on the Conduction of Heat by Liquids were marked by great experimental ingenuity. Among the most remarkable of his discoveries was what he called "Approach caused by Vibration," and his experiments on the discharge of electricity by the approach of hot bodies were not recognised as being of fundamental importance only because they were made about 20 years too soon.

To this Society he stands in the relation of practically sole founder. It owes its existence to a little circular he sent out on his own responsibility in the autumn of 1873. In the spring of 1874—just 40 years ago, therefore—the Society held its first meeting.

In addressing the Society, or reading a Paper, Guthrie's manner was sometimes almost portentously solemn, but in reality he was full of admirable humour. His strength of principle and the sacrifices he made to his sense of duty were not fully known during his lifetime, even to his most intimate friends. He died at the comparatively early age of 53, after a painful illness borne with uncomplaining fortitude.

All who knew Frederick Guthrie will agree that the Physical Society does well to keep alive his scientific reputation and to cherish his memory as a man.

XXIII. *Radiation of Gas Molecules Excited by Light.* *By* R. W. Wood, *Professor of Experimental Physics, Johns Hopkins University, Baltimore.*

Being the First Guthrie Lecture, delivered February 27, 1914.

The emission and absorption of light by molecules, and the closely related phenomenon of dispersion, have led us to the conception of something within the atom which is capable of responding to light-waves, in much the same way as a tuning fork responds to sound-waves of the same frequency as its own. There seems to be no escape from this conclusion, and very elaborate mathematical treatments have been built up on this foundation, which explain in a more or less perfect manner many of the phenomena in question. If, however, we try to form some conception of just what is going on we find that we are still very much in the dark. Helmholtz explained absorption by the introduction of a frictional term into his equations of motion for the atom, and though this led at once to an expression which represented anomalous dispersion, it left us in ignorance of how the energy absorbed by the molecules was transformed into heat, or, in other words, how the mean velocity of the molecules was increased by the excitation of vibrations within them. Planck avoided this difficulty by considering that the energy abstracted from the beam of light is re-emitted, though at the time the only experimental evidence which could be cited was the phenomenon of selective reflection, which occurs only when the molecules are so densely packed together as to give us the liquid or solid state.

What became of the absorbed energy in the case of a gas? This is something that I have been looking for for many years. Personally I do not require a working model, but I never felt *completely* satisfied by an equation in which absorption is represented by a frictional term or selective reflection predicted by the occurrence of an imaginary quantity, as in Lord Kelvin's expression for sodium vapour, especially as no case of such selective reflection by a vapour was known. The subject which I have chosen for discussion this evening is the disposition made by the absorbing mechanism of the energy which it abstracts from the incident light, and the more complicated phenomena which occur when various absorbing mechanisms

of the atom are coupled together. All of this has, of course, a very direct bearing upon the structure of the atom, a subject which is receiving much attention at the present time. Our profound ignorance of the matter and inability to construct or imagine a model capable of representing the source of even the simplest emission spectra, make one almost regret the enormous amount of work which has been spent in tabulating the wave-lengths of the spectra of the elements. It is only within the last decade that the brilliant work of Zeeman, Lorentz, Sir Joseph Thomson and others have led to a definite conception of the constitution of the outer shell of the atom. What is inside of the egg can only be imagined at the present time.

The problem of the structure of matter is one which must be attacked simultaneously from many sides, for it is improbable that any single weapon will cause the surrender of the secret. The spectroscope alone proved itself powerless, and the first definite step in advance was made when Zeeman placed a source of light in a magnetic field.

One great difficulty lay in the fact that in all known methods of exciting spectra it was "the whole or nothing." Flames, arcs, sparks and vacuum-tube discharges set in operation simultaneously a host of vibrations within the atom and resulted in a complex of lines. While it is true that much has been learned from the circumstance that the spectra vary according to the method of excitation, our ignorance as to the forces in operation in the case of flames or sparks makes it difficult to interpret the phenomena.

My own line of attack has been to keep the molecules as cool and quiet as possible, and then excite them to radiation by the application of an alternating electromagnetic field of a definite frequency, which is more commonly referred to as monochromatic light. We can in this case be pretty sure of what we are doing to the atoms, if we are not too particular to ask for a specific definition of what we mean by an alternating electromagnetic field.

That this method of going at the thing has simplified matters somewhat you will see when I draw your attention to sodium vapour which emits only one of the yellow D lines. Much time and many experiments have been necessary to develop the technique of exciting luminescence in this way, for the conditions vary according to the element studied, some vapours emitting light only when reduced to a pressure of less than 0·001 mm., while others operate even at a pressure of several atmospheres.

The presence of a foreign gas is very detrimental, which is sufficient to explain the failure of my first experiment on the subject made over 15 years ago, the concentration of sunlight by means of a large condenser upon a flame very rich in sodium.

In the earlier part of the work the phenomena presented were very complicated, and it is only recently that simple types of emission, which could be studied quantitatively, and subjected to mathematical analysis, have come to light. It will, therefore, be best to review the subject in almost inverse chronological order, beginning with the simplest case of a vapour which exhibits a single absorption line and emits radiations similar in every respect to the exciting radiations when stimulated by a frequency equal to that of the line of absorption. This condition is very perfectly fulfilled by the vapour of mercury, which has an absorption line at wave-length 2,536 in the ultra-violet.*

At room temperature the vapour of mercury has a pressure of about 0·001 mm. This gives us, assuming a uniform distribution, one molecule in every cube whose sides are equal to the wave-length of the ultra-violet light employed in the experiments. If a beam of monochromatic light of wave-length equal to that of the absorption line 2,536, obtained by isolating the corresponding emission line of the silica mercury arc by means of a quartz monochromator, was focussed at the centre of an exhausted quartz bulb containing a drop of mercury (the whole at room temperature), it was found that the light was powerfully scattered by the vapour, photographs of the bulb made with a quartz lens showing the cone of rays much as if the bulb were filled with smoke. The greater part of the light, however, passed through the bulb without sensible reduction of intensity even if the diameter of the bulb was sufficient to give to the luminous cone its maximum extension, for the cone is brightest where the rays enter the bulb, the intensity diminishing rapidly as we pass along the cone, owing to the removal from the incident beam of the energy of just the right frequency for exciting resonance. For a pressure of 0·001 mm. it was found that the intensity of the effective part of the incident beam was reduced to half of its original value after traversing a distance of 5 mm. in the vapour. This determination was made by passing a beam of *parallel* rays

* "Selective Scattering, &c., by Resonating Gas Molecules," "Phil. Mag.," May, 1912.

through the vapour, and measuring the intensity of the scattered radiation along the path. If the intensity at the point of entrance is 100, the intensity after traversing a distance of 1 cm. is 25, while at a distance of 2 cm. from the point of entrance the intensity is but 6. This means that in a bulb 3 cm. in diameter the luminous cone will scarcely reach the opposite wall. Notwithstanding the astonishing stopping power of this highly attenuated metallic vapour for waves of just the right frequency, we find that a large proportion of the energy passes through the bulb without being influenced by the vapour. Experiments showed that this was due to the circumstances that the emission line had a finite width, and that its central portion only was scattered by the resonating molecules. This is made clear by Fig. 1, in which ABD represents the intensity distribution in the emission line of the arc and cBc' the central portion removed by the mercury vapour

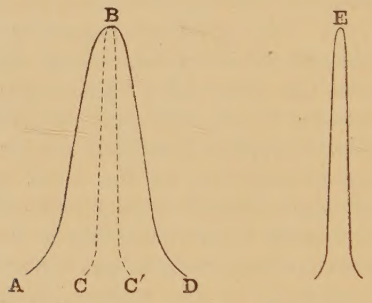

FIG. 1.

at room temperature. After passing through the bulb the emission line would appear furrowed by a fine black line of width cc' if we possessed a spectroscope of sufficient resolving power to show it. If the energy removed from the incident beam is re-emitted without change of wave-length by the vapour it is clear that the width of the spectrum line of the light given off by the cold mercury vapour in the bulb will be as shown by E in Fig. 1. In other words, we possess a method of obtaining light which is probably more homogeneous than any light obtained up to the present time. This is a matter which is being investigated quantitatively at the present time. Radiation scattered in this way without change of wave-length by resonating molecules I have named *Resonance Radiation*. We may term a bulb filled with vapour and emit-

ting light more highly monochromatic than the light which excites it a resonance lamp. With the light from a lamp of this description a photograph was made of a quartz bulb at room temperature containing a minute drop of mercury. The bulb appeared as if filled with ink, owing to the opacity of the mercury vapour for the rays. A drop of mercury placed on the top of a brass cylinder, heated to a temperature of about 5 deg. higher than that of the room, when photographed by the shadow method showed a column of vapour rising from the drop like black smoke. A lamp of this type has been used in many of the investigations of the scattering and absorbing power of mercury vapour, for the reason that it emits only waves which are in exact synchronism with the molecular resonators, whereas even the single line 2,536 isolated from the radiations of the mercury arc contains frequencies which are freely transmitted by the vapour.

Secondary Resonance Radiation.—It was found that the mercury vapour outside of the luminous cone traversed by the exciting rays also emitted light, a glow filling the entire bulb. Experiments showed that this was due chiefly to a secondary scattering of the light emitted by the directly excited molecules; in other words, the luminous cone of vapour acted as a light source which stimulated those portions of the vapour not actually traversed by the incident beam of light. The intensity of this secondary resonance radiation in comparison to that of the primary is surprisingly great, so great in fact that I was at first inclined to believe that it was due, in part at least, to a persistence in the luminosity of the rapidly moving molecules after they had passed through the region traversed by the exciting beam of light. Experiments showed, however, that the introduction of a thin quartz plate between the regions of primary and secondary resonance did not diminish the latter to any appreciable extent. This indicated that the phenomenon resulted from excitation by the light given out by the directly excited molecules, and did not result from a persistence of luminosity (phosphoresence), for the quartz plate is transparent to the ultra-violet light, but stops the moving molecules.

The comparatively great intensity of the secondary radiation results from the circumstance that at these low densities *no true absorption* exists; in other words, there is no transformation of energy. And now we come to a very important point, for we are at last in a position to measure the ratio of the scattered

to the absorbed energy, to define the conditions under which one or the other preponderates, and by varying the conditions to pass from complete scattering without absorption to complete absorption without scattering. Two methods have been found of making the measurements. We may compare the luminosity of the vapour with the luminosity of a surface of magnesium oxide when illuminated by the same beam of light. This is the direct method, and has been employed in the case of sodium vapour as I shall explain later on. Or we may measure the ratio of the intensity of the secondary resonance radiation to that of the primary. The value found for this ratio will show whether or not any true absorption has taken place, for it is possible to calculate the value of the intensity of the secondary radiation in terms of that of the primary resonance radiation if no true absorption exists. In making the experiments the conditions were made as simple as possible. A brass box was used furnished with windows of quartz, the

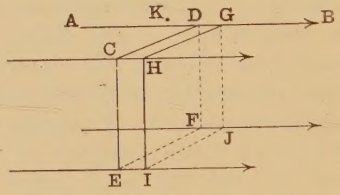

Fig. 2.

exciting beam of parallel rays of rectangular cross-section almost grazing the observation window. In this way it is possible to observe the primary and secondary resonance radiation through a minimum thickness of mercury vapour. Careful measurements showed that the intensity of the secondary radiation at a distance of about 1 mm. from the edge of the exciting beam was fully one-quarter of the intensity of the primary radiation observed within the beam of exciting rays. In some cases the value was very nearly one-third, but this was with an exciting beam of larger cross-section, and the explanation will be given presently. A rigorous calculation of the intensity of the secondary resonance under specified conditions is much to be desired, but the problem is a little more complicated than it appears to be at first, for each molecule is excited to a greater or less extent by the radiations from all of the other molecules. An approximation may perhaps be arrived at in the following way. We will consider

the incident beam AB, Fig. 2, of square cross-section (plane waves), and assume that all of the energy abstracted from it is re-emitted. We require the intensity of the secondary resonance radiation from a layer immediately above the plane CDGH of the thin cross-section of the exciting beam in comparison to the intensity of the primary resonance radiation within this same cross-section. The secondary resonance is excited by the energy poured out by the resonating molecules which lie in the path of the primary beam. The energy escaping from the rectangular cross-section figured above passes out through the four sides in equal amounts, for that which escapes through the surfaces CDEF and GHIJ remains within the beam and does not contribute to the excitation of the secondary resonance.

The excitation at a point, K, immediately above the exciting beam can, therefore, be regarded as dependent upon the energy stream passing through the side CDHG, which will be approximately one-quarter of the total energy radiated *laterally* from the cross-section figured. The total amount radiated laterally represents the energy abstracted from the primary beam by the molecules lying within the cross-section, and the intensity of the primary radiation will be proportional to this quantity. We should, therefore, expect the intensity of the secondary radiation to have about one-quarter of the value of the primary, as was found to be the case in the experiment. If true absorption exists the ratio will be very different. Suppose that, of the energy abstracted from the primary beam, one-half is re-emitted and one-half absorbed. The intensity of the primary radiation will now be only one-half as great as it was before. We can, however, raise it to its original value by doubling the intensity of the exciting beam. The amount of energy escaping from the sides of the cross-section figured is now the same as before, but only half of it is effective in producing secondary radiation, the other half being lost by absorption. The secondary radiation will now be only one-half as intense as it was before, while the primary will have the same value, the ratio being $\frac{1}{8}$ instead of $\frac{1}{4}$. We thus see that, by measuring the ratio of the intensities of the secondary and primary resonance radiation, we have a means of determining the ratio of scattering to true absorption. This makes a rigorous mathematical determination of the ratio which would be found in the case of complete scattering without any absorption much to be desired. The matter is peculiarly complicated,

for not only is a point lying outside of the primary beam illuminated by radiations coming from all points within the beam, but also by radiations from all points outside of the beam, and the same holds true for points within the beam. In other words, every molecule contributes, in a greater or less degree, to the illumination of every other molecule.

It would thus appear as if doubling the number of molecules in unit volume ought to more than double the intensity of the resonance radiation. This relation has not yet been investigated in the case of direct resonance radiation, but in the case of the fluorescence of iodine vapour, which is a more complicated phenomenon, a photometric study, made in collaboration with Mr. W. P. Speas, showed that doubling the vapour density by no means doubled the intensity. For example, we found that at a pressure of 3 the intensity was 24, while at a pressure of 6 the intensity was but 33. This is exactly the opposite of what is to be expected in the case of resonance radiation where all of the molecules are operating and there is no true absorption.

In the case of the iodine vapour, as the density of the vapour increases, the radiation from each molecule decreases as a result of the presence of its neighbours. The same effect is observed with a constant partial pressure of iodine, and an increasing pressure of some other gas, but iodine vapour is more effective in decreasing the luminosity of an iodine molecule than any of the other gases, and helium is the least detrimental, the gases arranging themselves in the order of their electro-negative character.

True Absorption.—The factor of true absorption makes itself manifest as soon as we admit air or some other foreign gas to the bulb containing the mercury drop. Even if the pressure is only a millimetre or two the effect is very marked. The intensity of the primary radiation is diminished to a certain extent, while that of the secondary radiation is reduced to a much greater degree for the reason mentioned a few moments ago. At a pressure of 6 mm. there is scarcely any trace left of the secondary resonance radiation, though the intensity of the primary is only reduced to one-third of its value in the complete absence of air, Fig. 3 (opposite page 201).

Huygens' Principle and Resonance Radiation: Selective Reflection.—Another point of considerable interest is the determination of the conditions under which it is possible to apply the principle of Huygens to the secondary waves emitted by

the molecular resonators. In certain theoretical discussions of absorption, the resonators are regarded as giving out waves which interfere destructively with the primary wave in the direction in which it is travelling, while in the opposite direction, there being no energy stream with which they can interfere, they unite into a wave which travels back towards the source, the phenomenon constituting selective reflection. While there is no doubt but that the resonators are close enough together to make the application of Huygens' principle justifiable, there are certain other factors which, it seems to me, must be taken into account.

In the first place, in a vapour at low pressure, the wave is obliged to pass by an enormous number of molecules before its intensity is much reduced. To give a numerical illustration, it was found that in the case of mercury vapour at room temperature the intensity was reduced to one-half its original value after the light had traversed a layer of the vapour 5 mm. in thickness. The pressure at room temperature is about 0·001 mm., which gives us about one molecule in every cube the sides of which are equal to the wave-length of the ultra-violet light. The light thus has to pass through 16,000 layers of molecules before losing one-half of its intensity, if we assume the molecules arranged in cubic order. If the radiations emitted by the molecules combined by Huygens' principle to form a regular wave, it would, in this case, constitute what we might term "volume reflection" as contrasted to surface reflection. It appears to me to be inconceivable that a reflection of this nature can occur, for the same reason that reflection cannot occur at the boundary between two media of different optical densities, if the transition is gradual instead of abrupt. The question may perhaps be raised as to why the molecules diffuse the light at all if there are many of them to the wave-length. We are accustomed to regard a medium in which the structure (so to speak) is small in comparison to λ as a homogeneous medium.

There is, however, in the present case another factor which doubtless has some bearing on the problem, namely, the circumstance that every molecule is excited to a greater or less degree by the radiations from its neighbours, all of which are moving at high velocities in all directions. This, it appears to me, would cause a random distribution of phase among the vibrations coming from the molecules, and would prevent

completely interference of the type considered in Huygens' principle.

Selective Reflection by a Dense Gas.—I have, however, observed that if the pressure of the mercury vapour is raised to several atmospheres regular reflection of a selective nature occurs at the inner surface of the bulb. The wave-length most strongly reflected is not quite in coincidence with the centre of the absorption line, but lies slightly on the short wave-length side of it. This is probably due to the circumstance that the refractive index of the vapour has an abnormally low value at this point, for the absorption line shows very strong anomalous dispersion. For a reflection from the inner surface of a quartz bulb we should expect a marked increase in the reflecting power for those values of λ for which the refractive index of the medium in the bulb was less than unity. The selective reflection of mercury vapour for wave-lengths in the vicinity of the 2,536 line should be investigated quantitatively. I have made only a qualitative investigation up to the present time, but as I have now learned how to control the intensity and width of the emission line of the mercury arc I feel certain that this can be done in a satisfactory manner.

I wish also to emphasise again the desirability of having a rigorous theoretical treatment of the emission of radiant energy, by resonating molecules, which give out again all of the energy which they abstract from the primary beam, both for a highly rarefied and a very dense gas. Much of the value of the experimental work will be lost if this is not done. Most important is the calculation of the relative intensity of the primary and secondary resonance radiation, under some condition verifiable by experiment. The best condition appears to me to be the case of an exciting pencil of parallel rays, of square cross-section, passing through the gas parallel to and almost grazing the window through which the observations are made.

Next in importance is the determination of what happens when the gas is dense enough to practically stop the incident rays before they have penetrated to a depth of more than one or two wave-lengths; if the intermediate condition can be examined, that too would be desirable. By this I mean the manner in which we pass from diffusion to regular or specular reflection, which I have examined experimentally. The regular reflection is not, however, nearly as powerful as we should

expect it to be if there were no absorption. As we increase the density of the vapour the intensity of the diffused radiation decreases and it is finally replaced by regular reflection. I do not, however, believe that the selective reflecting power is over 20 per cent., which means that the factor of true absorption has been introduced by the increase in the vapour density. I have not yet determined quantitatively the effect of increasing the density of the mercury vapour, as compared with the effect of raising the pressure in the same proportion by the introduction of some other gas. I am very sure, however, that true absorption is introduced to a greater degree in the latter case—that is to say, that collisions with foreign molecules are more effective in introducing absorption than collisions with mercury molecules. These questions I am now about to investigate; now that I am able to work with a source of constant intensity.

Case of Sodium Vapour.—Many of the phenomena which were discovered by photography in the case of the ultra-violet resonance of mercury vapour can be rendered visible by employing the vapour of sodium. The resonance radiation of this vapour I discovered in 1905,* heating the metal in an exhausted tube, illuminated by the rays from an oxy-hydrogen sodium flame brought to a focus by means of a large condensing lens. On gently heating the tube the path of the rays through the vapour was marked by a yellow glow, which drew back towards the side of tube through which the rays entered, as the vapour density increased, until only a thin skin of yellow light remained, which lined the inner wall of the tube, the image of the sodium flame appearing on the surface layer of vapour. Attempts to obtain regular selective reflection by increasing still further the vapour density failed, as a result of the chemical action of the vapour on the glass at high temperatures.

The experiment has been recently improved by L. Dunoyer, who employes a Bunsen flame charged with a spray of a solution of chloride of sodium, which is blown into the base of the burner by means of an atomiser. A chimney of sheet iron, with a square window, surrounds the flame, and an image of the window is projected upon the wall of the sodium bulb, by means of a quadruplet condenser, free from aberration if mono-

* Wood, "Phil. Mag.," August, 1905.

chromatic light is employed.* The bulb contains a little metallic sodium, carefully freed from hydrogen by distillation in vacuo, and is very highly exhausted. It is heated in a column of hot air rising from a tall chimney which surrounds a large Meker burner. The resonance radiation first appears at a temperature of about 125°C., a faint cone of yellow light appearing in the bulb. As the temperature is increased, it becomes brighter, and presently the secondary resonance radiation appears, a yellow glow filling the entire bulb. At a temperature of 180°C. the cone of primary resonance radiation disappears and with it the secondary radiation, and there remains only a brilliant square of yellow light on the front surface of the bulb, an image of the illuminated window thrown upon a resonating gas. It is as sharply defined as if thrown upon white paper, but has an intensity of only about one-tenth of the intensity obtained when a paper screen receives the rays coming from the condenser. In other words, the greater part of the light from the sodium flame passes through the vapour unhindered, the narrow cores only of the D lines being operative in causing resonance precisely as was the case with mercury vapour. If the bulb is allowed to cool slowly the first change observed is a fuzziness of the edges of the square image of the window, the appearance being as if the image suddenly went "out of focus." This is due to secondary radiation, and in its first stages it is confined to a shallow layer of vapour close to the wall. As the bulb cools down the cone of yellow light appears again, the phenomena previously described taking place in inverse order.

If, now, no absorption occurs in the case of sodium vapour, as appears to be the case with mercury vapour, we ought to obtain as brilliant an image on the vapour as on white paper, *provided our light source emits only light capable of exciting resonance.*

This subject has been investigated in collaboration with M. Dunoyer during the past winter. We fed the flame of the Meker burner with the spray of a solution of NaCl of varying concentration and measured the ratio of the diffusive power of a patch of MgO, formed on the outer wall of the bulb, to that of the diffusive power of the vapour within. It was found that the intensity of the resonance radiation from the sodium vapour was practically independent of the amount of

* L. Dunoyer "Sur l'Aberation de Sphéricité dans les Objectifs" ("Journal de Physique," 1913).

sodium in the flame, *i.e.*, of the total intensity of the source. This means, of course, that the *cores* of the D lines, which are alone effective in exciting resonance, do not increase much in intensity as we increase the amount of sodium in the flame. The ratio above referred to varied enormously however. With a solution made by diluting a saturated solution with 30 parts of water the ratio was 15 : 1—that is, the MgO was 15 times brighter than the surface layer of resonating sodium molecules in the bulb. With the solution diluted with 1,000 parts of water the ratio was 4 : 1 and with a still more dilute solution 3 : 1. The ratio could not be made smaller than this on account of the feeble intensity of the flame. The decrease in the ratio

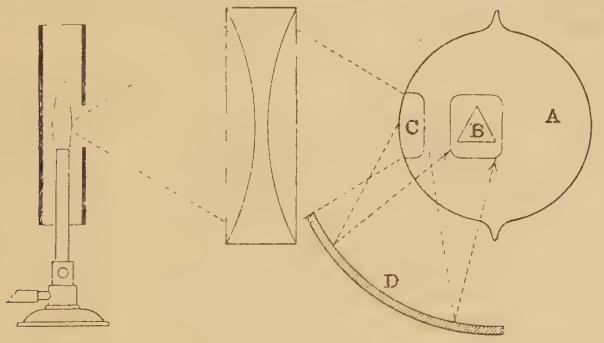

FIG. 4.

Experiment showing that the diffuse reflecting power of sodium vapour is equal to that of magnesium oxide for the highly homogeneous light of a sodium resonance lamp. A is a glass bulb, highly exhausted, containing sodium vapour. C is the image of the sodium flame thrown upon the vapour by the condensing lens. This image has an intensity of about one-fifth of the intensity shown by MgO under the same illumination. B is a triangular patch of MgO upon which an image of C is thrown by the concave mirror D. The MgO triangle cannot be distinguished from the luminous sodium vapour.

results from the circumstance that the width of the exciting lines (D lines) decreases as the amount of sodium in the flame is decreased; in other words, we are working with light more nearly in exact synchronism with the resonators. It is very important to know what the ratio would be if our exciting light were still more homogeneous. In this case we should have a ratio of 1 : 1, if the sodium vapour exhibits complete scattering with no trace of true absorption. It is impossible to accomplish anything in this direction by reducing further the amount of sodium in the flame, but by making use of the principle of the resonance lamp we can investigate the matter.

A single bulb was made to serve both as a resonance lamp and as a screen for measuring the reflecting power of the vapour, Fig. 4. An image of the sodium flame was thrown upon the side of the bulb and by means of a concave reflector, which had an effective aperture represented by F. 1 (formed by silvering one surface of double convex lens), an image of the spot of resonance radiation was thrown back upon the bulb of sodium vapour, one half of it being received by a small triangle of magnesium oxide, the other half by the vapour. Under these conditions it was found that there was absolutely no difference in the diffusive reflecting power. This means that the vapour of sodium at a pressure of probably less than 0·001 mm. has a reflecting power as great as that of the whitest paper or MgO, for light of exactly the right wavelength.

These experiments make it seem probable that absorption, as we usually understand the word, is a secondary action, resulting from reciprocal actions between the molecules. M. Dunoyer has made experiments on the effect of hydrogen on the intensity of the sodium resonance radiation and has found that the intensity is reduced practically to zero if hydrogen is present at a pressure of 10 mm. With helium at a pressure of half an atmosphere there is still considerable luminosity.

This indicates that the sodium molecule is far more sensitive to disturbances from neighbouring molecules than iodine, for which vapour the intensity of the fluorescence is reduced only from 100 to 35 by hydrogen at a pressure of 10 mm.

Now, the intensity of the iodine fluorescence decreases tremendously as the vapour density increases. From photometric observations made in collaboration with Mr. Speas I have calculated that the intensity of the radiation from an iodine molecule is reduced from 100 to 35 by a pressure increment of 0·25 mm. produced by increasing the density of the iodine vapour.*

Sodium is probably much more sensitive still, and the factor of true absorption is undoubtedly introduced by the reciprocal action between the molecules at even the very low pressure used in the experiments which have been described.

Thus far we have considered the type of resonance which results in the re-emission of radiant energy of the same type (*i.e.*, wave-length) as that of the exciting radiations. It has been studied for two cases, mercury vapour and sodium vapour,

* "A Photometric Study of the Fluorescence of Iodine Vapour." ("Phil. Mag.," March, 1914.)

the ultra-violet absorption (so-called) line of Hg at wave-length 2,536 and the D lines of sodium operating in this way. It appears probable that what is commonly spoken of as absorption results from some action upon the molecules of neighbouring molecules, for in both cases we find that an increase of vapour density or the presence of some chemically indifferent gas diminishes the resonance radiation and increases the factor of true absorption. It is to be clearly understood that by examination of the transmitted light we are powerless to discriminate between the two cases. The appearance of the spectrum is the same regardless of whether the molecules re-emit or absorb the radiations which they remove from the exciting beam.

The action of a molecule in destroying the resonance radiation of a neighbouring molecule appears to depend upon the electronegative quality of the molecule only. If the gas is strongly electronegative, resonance radiation only appears at very low pressures (bromine) or not at all (chlorine). If less strongly electronegative (iodine) we have radiation even when the pressure amounts to several millimetres, while with an electropositive molecule (mercury) resonance radiation persists even when the pressure is as high as several atmospheres.

Of all the gases, helium appears to be the least destructive in its action upon molecular radiation; at least this is true for iodine vapour, and M. Dunoyer and I have recently found that it is possible to have a fairly bright resonance radiation of sodium in helium at a pressure of half an atmosphere.

The addition of a chemically indifferent gas not only changes the phenomenon of resonance radiation into absorption, but also increases the width of the absorption line as observed with a spectroscope. The line becomes fuzzy and is less black at the centre. I have not yet examined the action of different gases upon the absorption lines to see whether helium at, say, 50 mm. and CO_2 at 2 mm. have the same effect upon the absorption lines, which are the pressures which produce the same destructive action upon the fluorescence in the two cases. It is my belief, however, that the electronegative quality of a gas will not have much effect upon the change produced in the appearance of the absorption lines.

We now come to the second part of our subject. In many cases the molecule, when excited by monochromatic radiation, emits not only radiation of this same wave-length, but also

other wave-lengths which form what I have named Resonance Spectra. It seems as though there were numerous vibrating systems in the atom, the excitation of one system being communicated to the others in some way. Even the mercury 2,536 line and the D lines of sodium cannot be considered as due to a simple isolated vibrator, for I have found that mercury vapour can be caused to emit the 2,536 line when excited by extremely short ultra-violet waves, below wave-length 2,000 in the spectrum, and sodium vapour can be caused to emit a yellow band at wave-length 5,890 when excited by blue-green light.

We will consider first an experiment which I suggested in the Paper of 1905 already referred to, and which I have just brought to a successful conclusion in collaboration with M. Dunoyer. The question which we wished to solve was whether the mechanisms which produce the D lines are connected ; in other words, if sodium vapour is excited to radiation by monochromatic light of the wave-length of D_2 only, will it emit light of the wave-length of D_1 as well as that

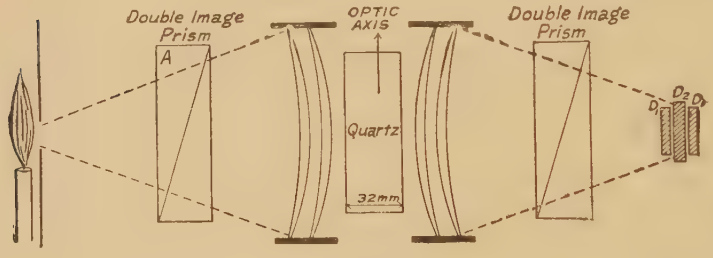

Fig 5

of the exciting radiation (D_2)? For this experiment we require some means of obtaining a powerful beam of light containing only the radiations of one of the two sodium lines. This was accomplished by means of a device which I described many years ago, depending upon the double refraction of quartz.

I have since improved this polarisation method, so that practically no light is lost (*see* Fig. 5).

Double-image prisms are used instead of nicols, and the two images which contain only D_2 light are united by a suitable rotation of the second prism, the D_1 images lying to the right and left. The separation of the two wave-lengths is effected by a block of quartz 32 mm. in thickness, cut parallel to the

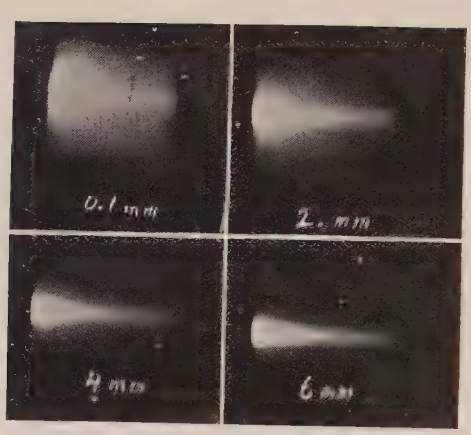

FIG. 3.

Cone of primary resonance radiation surrounded by the glow of the secondary resonance radiation. The latter has disappeared in the fourth picture as the result of admitting air at a pressure of 6 mm. "Phil. Mag.," May, 1912.

FIG. 6.

To face page 201.]

axis, which causes the two D lines to emerge polarised at right angles to each other. The monochromatic light obtained in this way is represented in Fig. 6, in which the upper portion of the slit is illuminated by D_2 light and the lower by the light of the sodium flame.*

With this apparatus we succeeded in exciting a bright resonance radiation of the sodium bulb by the light of D_2, and with an exposure of five hours obtained the spectrum of the emitted light, a single line only (D_2) with no trace of D_1. *It is thus possible to have sodium vapour emitting one only* of the D lines, and the experiment is a very good illustration of how spectra may be simplified by exciting them by means of light-waves.

We have also photographed the spectrum of the resonance radiation of sodium vapour excited by the two D lines, and have found that the emitted lines have practically the same intensity. Now, it is well known that in the sodium flame (which was used for the excitation) D_2 is much stronger than D_1, and it is very surprising to find that this same difference does not appear in the case of the resonance radiation. I have made a study of the relative intensity of the two lines in the case of flames, and have found that, with a flame very feebly coloured with sodium, D_2 is fully four times as intense as D_1. With a flame very heavily charged with the vapour D_2 is certainly not over 1·3 times as bright as D_1. Two photographs were made in coincidence simultaneously, with an exposure of half an hour, the light of the very intense flame passing through a piece of densely smoked glass before entering the instrument. The light from the feeble flame was reflected by the comparison prism. The smoked glass was so chosen that the intensity of the bright flame was reduced to almost exactly that of the feeble flame. To obtain the large ratio it is necessary to use a flame much less coloured than the flames commonly employed.

In the case of the D lines there seems to be no connection between the two vibrating systems, and I have never been able to get the D lines by ultra-violet excitation at the point of the spectrum occupied by the second members of the principal series. It may, however, be possible to do this with the improved methods.

* A full description of the apparatus will be found in the "Phil. Mag." for March, 1914, from which Figs. 3 and 6 are taken.

We will now examine some cases in which we have a connection between the vibrators.

Resonance Spectra.—It will be impossible to give anything more than a very brief outline of the extremely complicated relations which exist in the case of these remarkable spectra, which appear to become more and more involved the more we study them. A very complete account of what is known at the present time regarding the case which has received the fullest study, will be found in the "Philosophical Magazine" for November, 1913, and the "Phys. Zeit," December 1, 1913.*

The discovery of resonance spectra was made in the course of some experiments on the fluorescence of sodium vapour. It was found that if the vapour was illuminated with monochromatic light of wave-length corresponding to that of one of its many thousand absorption lines, it emitted a spectrum consisting of widely separated lines spaced with remarkable regularity along the spectrum. The phenomenon is exhibited only by sodium vapour of considerable density, *i.e.*, at a pressure of the order of magnitude of a millimetre or two. Its absence in the case of vapour at such low pressures as are used for the study of the resonance radiation (D line emission) may be due to its relative faintness, or perhaps to the circumstance that it results from molecular aggregates which do not form at the low pressures. This is a point which will probably be settled by some investigations now in progress.

The absorption spectra of iodine and bromine are very similar to that of sodium. With my plane grating spectrograph at East Hampton, the focal length of which is nearly 14 metres, I have found that there are about 35,000 absorption lines in the visible spectrum, 115 having been counted in a region no wider than the distance between the D lines of sodium.

Now, since we obtain a different emission spectrum for each absorption line which we excite by monochromatic light, we are confronted with the case of an element which can theoretically be made to emit *many thousands of spectra, no two of which are precisely alike.* The number of spectra possible is vastly in excess of the total number of absorption lines, for we may excite two or more lines simultaneously, in all possible combinations, so that *the total number of different spectra possible*

* Wood, "Resonance Spectra of Iodine under High Dispersion."

is practically infinite. In fact, it is only by taking special precautions that a single absorption line can be excited, for the lines are so close together that most of the emission lines of metallic arcs, which are used for the excitation of the vapour, have a natural width of such magnitude that they cover from two to a dozen of the absorption lines. The study of these resonance spectra has appeared to me to be of the greatest importance, for it is practically the only case in which we have any exact knowledge of the nature of the excitation. In flame, arc and spark spectra we know practically nothing about the forces which are at work upon the molecule, while in the case of resonance spectra we can be reasonably sure that we are subjecting the molecule to alternating electromagnetic forces of a definite frequency, and nothing else. Of course, these forces may bring other factors into play by dissociating the molecules or breaking up molecular aggregates, though I regard the fact that the emitted light is strongly polarised as evidence against this. Experiments made in Prof. Pupin's laboratory at Columbia University have shown that if an armature, free from current initially, is rotated in an alternating magnetic field the armature will deliver a current made up of a large number of different frequencies increasing by constant increments, and furnishing an interesting analogy with the resonance spectra.

I will now give a brief résumé of the results which have been obtained with iodine vapour, omitting most of the experimental details, which are very fully described in the Papers already referred to.

The vapour is used at room temperature in highly exhausted tubes, the excitation being by the radiations from a quartz mercury arc, sorted out by suitable ray-filters. The most careful study has been made in the case of the excitation by the green line (5,461), which can be made to cover from one to 10 absorption lines, by varying the watt consumption of the lamp. By interposing a ray-filter of bromine vapour it is even possible to remove some of the frequencies of the broadened green line, and so "throw out of action" the iodine absorption lines which happen to coincide with the bromine lines. The resonance spectra excited in this way have been photographed in the fourth order spectrum of a 6 in. plane grating used in conjunction with a collimator and a specially constructed Cooke portrait objective of about 130 cm. focus.

The vapour of iodine was contained in long glass tubes,

blown out to a small bulb at one end, an image of the mercury arc being formed along the axis by means of a large condenser.

It was found in the earlier work, that if the resonance spectrum, excited by the green mercury line emitted by the quartz arc operating under normal conditions, was photographed under high dispersion, the resonance lines were resolved into close groups. This was found to result from the circumstance that the green line was wide enough to cover seven absorption lines. By operating the lamp at a lower temperature—that is, with a small voltage drop across its terminals—the width of the green line can be reduced until it covers but a single absorption line, or at most two. The resonance spectrum is now found to be much simpler, the complicated groups being replaced by single lines or by pairs of lines. By raising the voltage of the exciting lamp, the number of lines in the groups can be gradually increased, for the green line broadens and covers other absorption lines as the watt consumption increases.

The groups are spaced at nearly equal distances along the normal spectrum, the distance between them increasing by a nearly constant amount as we pass from group to group towards the red end of the spectrum. There are slight variations, however, from the law of constant second difference, which cannot be explained by errors of measurement, for the wave-lengths have been determined to within 0·02 A.°E. in the case of the photographs made in the fourth order spectrum.

In my last Paper I gave a table of wave-lengths for the lines in all of the groups for the excitation by the broadened green line. The wave-lengths for the single lines and pairs, which replace the groups, when the excitation was by the narrowed green line (covering one absorption line only), were recorded on the photograph only, and the differences and second differences were not given. It may, therefore, be well to record them here in the form of a table.

The study of this comparatively simple spectrum is somewhat complicated by the circumstance that at two or three points we have single lines instead of pairs, and one or two of the pairs are made up of components of very unequal intensity, the components being a little closer together than is the case with the majority of the pairs. The spectrum which we are now considering is reproduced in Fig. 1, Plate XV., of the Paper referred to above. Some of the fainter lines result from excitation by the yellow mercury lines, which were not screened

off in this case. Two pairs in the series are missing altogether, and I have divided the difference in λ between the adjacent pairs by 2 in these cases.

The left-hand table gives the wave-lengths of the longer wave-length components of the doublets, the right-hand table the components of shorter λ. Absent members are indicated thus ....., and in the case of the single lines I have placed them in the left-hand table as this appears to be their proper place.

λ	*Differences.*		λ	*Differences.*
6560·7	—		6558·4	—
.....	82·0		.....	82·0
6396·3	—		6394·3	—
—	79·7		—	79·9
6316·6	—		6314·4	—
—	78·6		—	78·3
6238·0	—		6236·1	—
—	77·1		—	77·0
6160·9	—		6159·1	—
.....	75·0		.....	75·0
6010·8	—		6009·1	—
—	73·0		—	73·9
5937·6	—		5935·2	—
—	71·7		—	70·7
5866·1	—		5864·5	—
—	70·1		.....	69·1
5796·0	—		—	—
—	69·4		.....	—
5726·6	—		5657·1	—
—	67·8		—	67·6
5658·8	—		5589·5	—
—	66·8		—	64·5
5592·0	—		5525·0	—
—	65·5		—	64·3
5526·5	—		5460·7	—
—	65·8		—	—
5460·7	—		—	—

As will be seen from the table, the second differences vary in an irregular manner. The doublets are very clearly defined in the red and orange of the spectrum, but in the region between the green and yellow mercury lines they are not so pronounced, being replaced by single lines in two cases, and by a single line with a fainter series on the short wave-length side in the case of the one at 5726·6.

Multiplex Excitation.—By raising the terminal voltage of the lamp (by gradually cutting out the resistance in series with it) other lines make their appearance to the right and left of each doublet, until we finally have groups containing as many as a dozen lines. This occurs when the green mercury line has covered eight iodine absorption lines.

The total width of each group of lines is about 30 times as great as the width of the group of absorption lines covered by the mercury line. The groups in the immediate vicinity of the exciting lines are very similar in appearance, and the regularity of their disposition along the spectrum reminds one of the diffraction spectra exhibited by a grating of small dispersion. For convenience in referring to them we may designate them as groups of the +1st, +2nd, +3rd, &c., *orders*, adopting the same nomenclature as in the case of grating spectra. Those on the short wave-length side of the exciting line we may designate −1st, −2nd, −3rd orders.

I have observed as many as 20 orders on the red side (*i.e.*, + orders) and two or at most three on the short wave-length side (*i.e.*,− orders). These latter orders constitute exceptions to what is known as Stokes's Law. In the case of the excitation by the green line the −order groups are extremely faint, while in the case of the excitation by the yellow lines they are very strong. In other words the exception to Stokes's law become more conspicuous as we excite with vibration of lower frequency. The same thing was found in the case of sodium vapour.

Origin of the Groups.—The key to the question as to how the groups originate lies in the fact that a group appears at the point of the spectrum occupied by the exciting line. This group we may call the group of zero order, and as I have already said it is about 30 times as wide as the group of eight iodine absorption lines, which we are stimulating and which lie at the centre of the zero order group.

It is a little difficult to express in words the rather complicated phenomena involved. In the Paper referred to I have attempted to account for the formation of the groups, but as explanation may be found somewhat long-drawn-out and difficult to follow, I will try to give it in a simplified form in the present report. Let us suppose that we are stimulating three iodine absorption lines instead of eight. These lines are the lines 1, 2 and 3 of Fig. 7. They are so close together that the spectroscope used for photographing the resonance spectra could not possibly resolve them. The width of the group of lines 1, 2, 3 and A, B, C, should be about $\frac{1}{30}$th of the distance between B′ and A′ to have the diagram in correct scale. The stimulation of absorption line 1 causes the vapour to emit the same wave-length which gives us line A (immediately below

line 1 in the diagram). This is what we call resonance radiation, and we may designate this line as the RR line, The vapour emits in addition the lines A of 1st order group and A of 2nd order group, &c. If this were all, we should have a resonance spectrum of the simplest type, consisting of a series of equidistant single lines.

In the earlier investigations I was of the opinion, that resonance spectra were of this nature, but it is now certain that such is not the case, for we should have, under these conditions, each of the absorption lines 1, 2 and 3 giving a series of equidistant lines, which, if the spacing were the same, would coincide, while if the spacing were different for the three series arising in this way we should have groups of three lines each, similar in appearance but with their components separated by increasing amounts as we pass from group to group. There would, in this case, be no group but only a single line (in reality three close unresolved lines at the position of zero order).

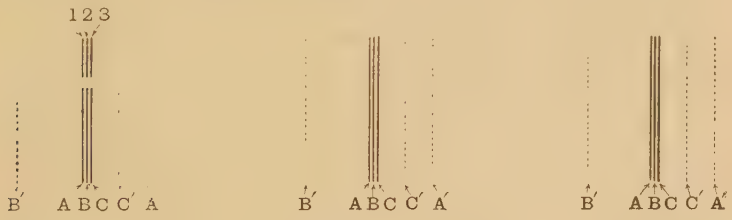

Fig. 7.

I am forced to the conclusion that absorption line 1 gives us, in addition to the lines A, the companion lines A′. Absorption line 2 gives us the lines B and the fainter companions B′, which we will assume to lie to the left of the strong lines in this case.

In the same way absorption line 3 gives us the lines C and the companions C′, the companions, in this case, lying to the right of, and nearer to, the main line. The main lines A, B and C of each group appear superposed with the resolving power obtained in the first order spectrum of the grating, and form what I termed the core of the group in the Paper referred to. In the photographs made in the 4th order spectrum they appear slightly separated, showing that the spacing of the three series is not quite the same. This explanation accounts for the similarity in the appearance of the groups, the fact that the distance between the components is essentially the same,

and the fact that we have a similar group (zero order) at the position of the exciting line.

In the communication to the "Philosophical Magazine" there will be found a diagram of the changes which occur in the groups resulting from the addition of new companion lines as the broadening mercury line covers more and more of the iodine absorption lines.

One of the most important questions, which I hope to solve next year, is whether the nature of a resonance spectrum is the same when the exciting line falls on the right-hand edge of an absorption line as when it falls on the other edge; in other words, when the frequency of the light is very slightly greater or less than the frequency of the absorption line.

Polarisation of the Light of Resonance Spectra.—The circumstance that the light emitted by the iodine vapour is strongly polarised, and that the polarisation is equally marked in all of the lines of the resonance spectrum, indicates that the emission of light is the direct result of vibrations forced in the molecule by the light waves, and does not depend upon dissociation and recombination. A short communication of this subject will be found in the "Phil. Mag." for November, 1913, illustrated by photographs of the resonance spectrum crossed by the Savart fringes, which give the measure of the polarisation.

Radiation of Gases produced by Ultra-Schumann Waves.—We come now to another side of the subject of which I should like to make brief mention.

Several years ago I made some experiments with a view of ascertaining whether the absorption of Schumann waves by air or oxygen was accompanied by an ultra-violet flourescence. A powerful condenser spark was allowed to play against the under side of a metal plate perforated with a small hole* at the point against which the spark played. It was found that if a photograph of the region above the plate was photographed in a dark room, with a quartz lens, a luminous "squirt" appeared in the pictures issuing from the hole. The spectrum of the squirt showed that the light was made up chiefly of the so-called "water band" of the oxy-hydrogen flame, together with a number of the stronger nitrogen bands, *with no trace whatever of any of the spark lines*, showing that the "squirt" was not the light of the spark diffused by dust in the air. It

* "A New Radiant Emission from the Spark." "Phil. Mag.," Oct., 1910.

was found to be much more intense in nitrogen than in air, and much less intense in oxygen. If the hole was covered with a very thin (1 mm.) plate of fluorite, however, all trace of the squirt disappeared, which made it appear as if the luminosity must be produced by some emission from the spark other than the Schumann waves, which are transmitted by lenses and prisms of fluorite.

During the past winter the subject has been investigated again in collaboration with G. Hemsalech and many new and very remarkable phenomena have come to light. In the first place we have found that even a rather feeble air blast across the "squirt" destroys its luminescence at the spot traversed by the jet, though it is quite as bright both below *and above* the jet as before. This shows that *air in motion* does not become luminous. We are unable to explain this phenomena to our complete satisfaction; at first we thought that the air might remain luminous after the passage of each spark (phosphorescence) as in the beautiful vacuum tube experiments made by Strutt, the action of the air-blast being the continuous sweeping away of the luminous gas, but long exposures, with feeble air jets, failed to show any evidence of luminosity in the air jet after its passage across the squirt.

With nitrogen, however, the reverse is true. The luminosity is much more intense (perhaps 10 times) in the jet, and the spectrum of the light from the moving jet shows the nitrogen bands, with no trace of the "water-band," which predominates in the "squirt" (in air) above and below the moving jet of nitrogen. If oxygen is added to the nitrogen the luminosity becomes much less, which agrees with the powerful influence of oxygen in destroying the fluorescence of iodine. If we place a quartz prism in front of the quartz lens, in making our photographs we find that the luminosity in the moving nitrogen jet is displaced to one side of the monochromatic image of the squirt (water-band image). This displacement results from the difference of the wave-length of the light emitted by the nitrogen (the nitrogen bands). Similar displaced images were obtained with jets of hydrogen, carbon dioxide and coal-gas, each gas giving a characteristic spectrum. The phenomena are too complicated to be given in much detail in the time at my disposal and I shall have to refer you to the Paper published by Mr. Hemsalech and myself which will appear shortly.*

* "Phil. Mag.," May, 1914.

I have still more recently endeavoured to find some vapour or gas which would emit visible radiations when passed through the squirt, as the phenomena could be much more satisfactorily investigated if we were not obliged to work always in the dark with photographic methods. The first substance which I tried was iodine, and as the detrimental effect of oxygen was well known the vapour was carried across the squirt in a jet of nitrogen. A few iodine crystals were placed in the glass tube which delivered the nitrogen jet to the "squirt." Even at room temperature a faint bluish-green fluorescence was visible, which became quite bright if the tube was slightly warmed.

If air was used instead of nitrogen, the iodine vapour refused to respond.

A fluorite plate was now placed over the hole above the spark, and the green fluorescence at once appeared, but faded away very rapidly, disappearing entirely in about 15 seconds. The fluorite plate was moved a trifle and the fluorescence at once appeared again. It was at once apparent that the spark vapours formed a deposit on the fluorite plate which made it opaque to the rays which excited the fluorescence.

It is rather remarkable to find iodine vapour fluorescing brightly when mixed with nitrogen at atmosphere pressure when we remember that the fluorescence excited by blue-green light is reduced practically to zero by nitrogen at a pressure of only 8 cm. I have not yet examined the spectrum of the iodine vapour excited by the spark rays. The colour of the light is bluish-green, in contrast to the yellowish-green fluorescence excited by visible light, and it is evidently much less affected by the presence of a foreign gas. The discovery of the deposit formed upon the fluorite plate suggested at once that the failure to observe any trace of the "squirt" in air (even with long exposures) when the fluorite was used was in all probability due to the formation of this deposit.

I accordingly repeated the experiments with air and nitrogen, cleaning the fluorite plate every 15 seconds, and succeeded in getting the "water-band" squirt, as well as the one of nitrogen. It was found, however, that it was necessary to give an exposure about 80 times as long as that necessary without the plate to obtain an image of equal intensity. This indicates that a plate of fluorite 1 mm. thick transmits only a trifle over 1 per cent. of the energy of the radiation which excites these gases to ultra-violet fluorescence. The waves are thus undoubtedly shorter than those discovered by Schumann, since these pass readily through lenses and prisms of fluorite.

Attempts are being made at the present time to develop a method of measuring, at least roughly, the wave-length of these radiations.

In conclusion, let me say that I have endeavoured to show what has been learned from the study of these vibrations, forced in the molecule by means of light waves.

It is my hope that the study of resonance spectra and related phenomena may, in time, furnish one number in the "combination" necessary to unlock the secret of molecular radiation.

XXIV.—*A Graphic Treatment of Cusped Wave-fronts and of the Rainbow.* *By* WILLIAM R. BOWER, *B.Sc., A.R.C.S., Technical College, Huddersfield.*

CONTENTS.

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§ 1. *Introduction.*

A method of drawing the cusped wave-fronts produced by refraction and reflection at a spherical surface, and a graphic treatment of the elementary theory of the rainbow are described in the Paper. The properties of the Centre of Junction discovered by Young (Phil. "Trans.," 1801, XCI., p. 23) and independently by Cornu ("Science Abstracts," V. (1902), 1835) form the basis of the constructions.

§ 2. *Refraction by a Spherical Surface. Position of the Junction-centre.**

(I.) In Fig. 2, when BY, BY′ are the incident and refracted directions of the chief-ray of a pencil and CY, CY′ are drawn

* An account is given in a Paper by Mr. W. R. Bower on "A Graphic Method of Optical Imagery," Proc. Phys. Soc., Lond., Vol. XXV. (1913), pp. 171–176.

from the centre C perpendicular to BY, BY′ respectively, then $CY=\mu . CY'$. CKZ′Z and YY′K are at right angles; K is the junction-centre and Z, Z′ the aplanatic pair of points for the chief ray. Also $CZ=r . \mu$ and $CZ'=r/\mu$.

Draw KF′ parallel to BYZ. Then F′ is the second principal focus of the small pencil of meridian rays associated with the chief ray.

(II.) As B moves further from A—that is, as the angle of incidence φ increases—so does the angle of refraction φ', also $\varphi-\varphi'$, and, therefore, the angle BG′C.

Because YK and CZ are always perpendicular, K and F′ rise respectively from C and G′; finally F′, K, Y′, Z′ are coincident. Y then coincides with B and YY′ is tangential to the circle of radius r/μ.

(III.) Write y, y', f, f' for BY, BY′, BF or F′K and BF′ respectively. Then, since the triangles βZC, Y′CY are similar,

$$\beta Z/CZ=Y'Y/CY.$$

Also, since the triangles YZC, KYC are similar,

$$CZ/YZ=CY/KY.$$

$$\therefore \quad YZ/\beta Z=KY/Y'Y=BF'/BY',$$

or

$$YZ/\beta Z=f'/y'.$$

§ 3. *Reflection by a Spherical Surface. Position of the Junction-centre.*

(I.) Let (Fig. 1) the reflected portions, BY′, B_1Y_1', of two parallel rays, BY, B_1Y_1, intersect at E′. Draw CY, CY′ perpendicular to BY, BY′ respectively. Then k, the intersection of YY′ and BC, is the junction-centre for the chief ray YBY′.

(II.) The angle $BE'B_1$=angle YBE′—angle Y_1B_1E'. The angle BCB_1=angle YBC—angle $Y_1B_1C=\frac{1}{2}$ angle $BE'B_1$. Then the angles $BE'B_1$, $BC'B_1$ are equal, and B, B_1, E′, and the mid-point, C′, of BC are concyclic.

(III.) As B_1 moves up to B, the locus of E′ is the fixed line BE′. When B_1 is very close to B, the angle BB_1C' becomes sensibly a right angle. Also the angle BE′C′.

Then E′ sensibly coincides with the mid-point, F′, of BY′.

Thus F′ is the second principal focus of a small pencil of meridian rays incident at B.

Also F′k is parallel to BY; and F, the mid-point of BY, is the first principal focus.

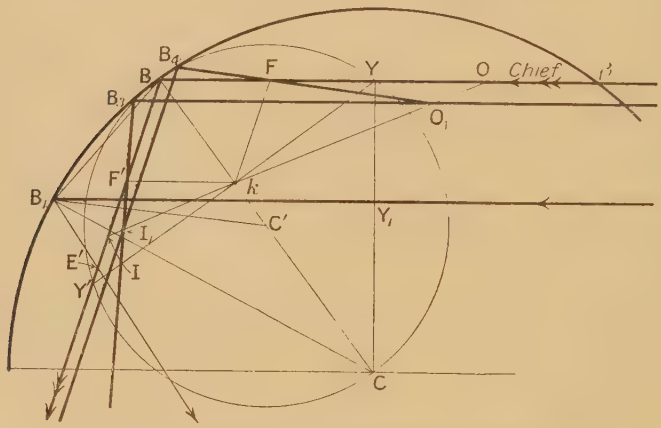


Fig. 1.—Reflection by a Spherical Surface.

(IV.) *To prove that the line joining an object-point, O, and its meridional image-point, I, passes through the junction-centre, k.*

A ray parallel and close to BY, incident at B_3, is reflected through F′. A ray, O_1F, from any object-point, O_1, on the line B_3O_1, incident at B_1, is reflected parallel to BF′. Let B_1I_1 and B_3F' intersect at I_1. I_1 is the primary or meridional image of O_1.

Join I_1O_1 and I_1k. Then sensibly

$$\frac{B_3O_1}{BF}=\frac{B_3B_1}{BB_1}=\frac{I_1B_3}{I_1F'},$$

$$\therefore \quad \frac{B_3O_1}{I_1B_3}=\frac{F'k}{I_1F'}.$$

∴ the triangles $I_1F'k$, $I_1B_3O_1$ are similar. Then k is on the line I_1O_1.

When B_3O_1 is very close to BY, O_1 coincides with O and I_1 with I. Thus the position of the meridional image-point on the reflected ray is collinear with the junction-centre, k, and the object-point, O.

(The position of the junction-centre may also be inferred from the general construction (§ 2 (I.)) by putting $\mu=-1$.)

(V.) Since $\frac{BF}{BO}=\frac{F'k}{BO}=\frac{IF'}{IB}=\frac{IB-F'B}{IB}$ $\therefore$ $\frac{BF}{BO}+\frac{BF'}{BF}=1$;

and since $BF=BF'=BY/2$,

$\therefore$ $1/BO+1/BI=1/BF'=4/B\beta=2/r.\cos\varphi$.

(VI.) When O is on the circumference, $BI=B\beta/3$.

§ 4. *Distribution of Foci of a Small Incident Meridian Pencil of Parallel Rays.*

(I.) In Fig. 2 let the chief ray be incident at B. Find the junction-centres, K, k', k'' . . . between the incident and

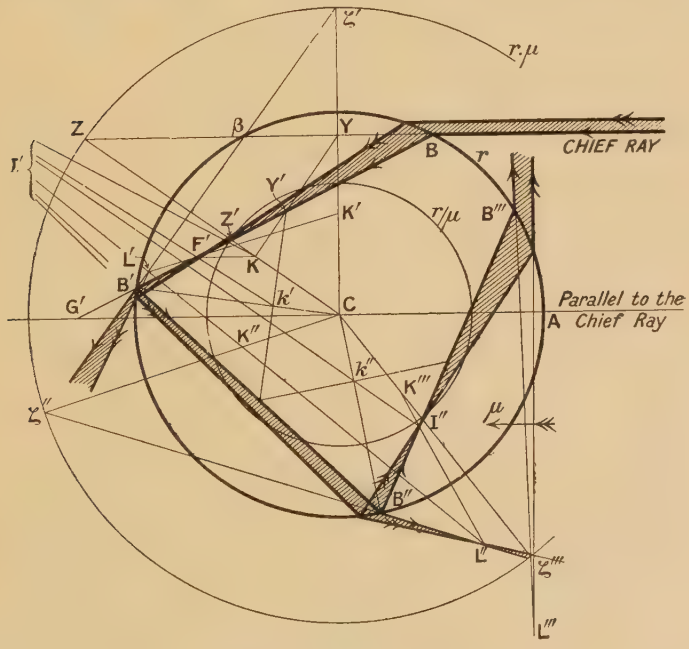


Fig. 2.—Positions of Foci.*

refracted parts, the refracted and first reflected portions, the first and second reflected portions, &c.

* In this figure some points and lines (especially CY, Cζ'') appear coincident that are not necessarily so. K''', I'', L''' are collinear.

Find F′ by drawing F′K parallel to the incident ray then I′, I″ . . . by drawing F′k′I′ and I′k″I″, &c.

The points, L′, L″, L‴ are the image-points on the emergent pencils at B′, B″, B‴. They are obtained from F′, I′, I″ through the junction-centres, K′, K″, K‴, &c. Make B′ζ′ =BZ, &c. The points, K′, &c., are obviously obtained on Cζ′, &c., by making CK′, &c., equal to CK.

(II.) Suppose (Fig. 3) the parallel pencil, whose chief ray is

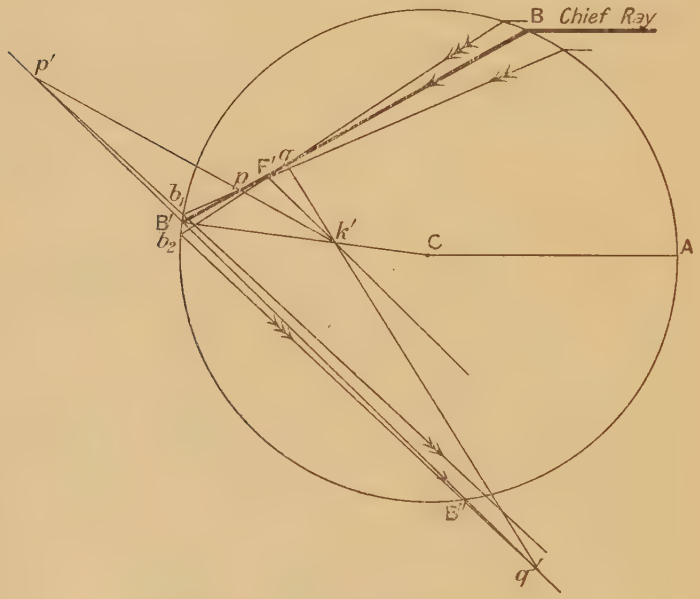


Fig. 3.—Parallel Reflected Pencil.

incident at B, is reflected at B′ as a parallel pencil. Then F′k′ is parallel to B′B″.

A neighbouring ray incident between A and B intersects BB′ at p between F′ and B′, and the spherical surface at b_1 above B′. The line, $k'p$, intersects B″B′ at p' to the left of B′: then $p'b_1$ is the direction of the reflected ray.

A neighbouring ray incident above B intersects BB′ at q between F′ and B and the spherical surface at b_2 below B′. The line $k'q$ intersects B′B″ at q' to the right of B″: then $q'b$ is the direction of the reflected ray.

§ 5. *Cusped Wave-fronts.**

(I.) The wave-fronts after refraction or reflection at a spherical surface may be readily drawn when the rays and loci of the focal points have been obtained, as in §§ 2, 3, 4.

Referring to Fig. 4, the points 1′, 2′, 3′, &c., are the meridional foci of the refracted pencils produced from incident parallel pencils. The point of incidence of (5) is taken (§ 8), so that the refracted pencil through 3′ is reflected as a parallel pencil. Also (1) is incident tangentially at B_1 and (8) normally

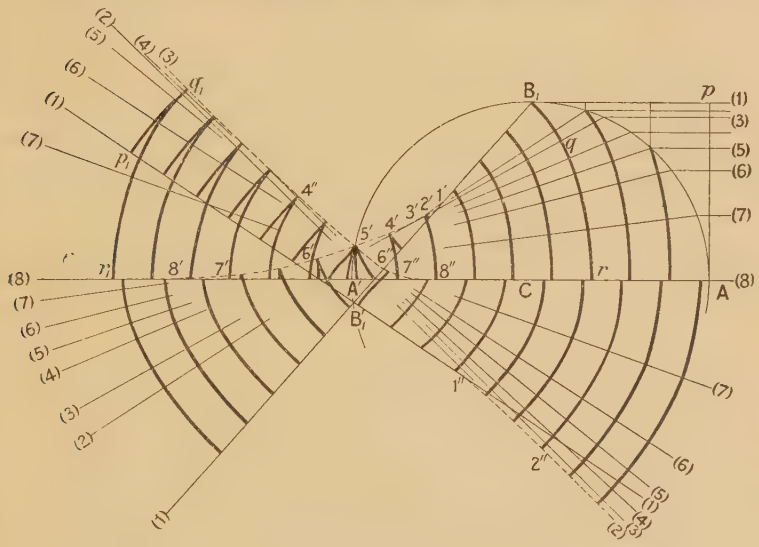


Fig. 4.—Wave-fronts on Refraction and Reflection.†

at A. Then the caustic from 1′ to 8′ is the locus of the meridional foci by refraction of the parallel pencils incident between B_1 and A.

Consider the reflection of the pencils through 1′, 2′, &c., at

* *See* a Paper by Prof. W. B. Morton, on "Cusped Waves of Light and the Theory of the Rainbow," *Proc.* Phys. Soc., Lond., Vol. XXIII. (1910), pp. 58-64. In this other references are given.

† In Fig. 4, the wave-fronts in the first and third quadrants are those of the plane wave refracted at the upper part of the spherical surface. The wave-fronts in the fourth quadrant are those of the wave after reflection by the spherical surface. The wave fronts in the second quadrant are continuous with these. In the fourth quadrant the wave-front between the caustic through 1″, 2″ and ray (1) is practically coincident with the wave-front between the caustic and rays (4), (5), &c.

points on the spherical surface. The points 1″, 2″, &c., are conjugate foci of 1′, 2′, &c., by reflected meridional rays : 5″ coincides with 5′, 3″ is at an infinite distance. The conjugate foci of the points on the caustic by refraction between 1′ and 3′ are on the branch 1″, 2″, 3″ (at infinity) of the caustic by reflection, and of the points 3′ . . . 8′ on the branch, 3″ (at infinity), 4″ . . . 8″. The caustic by reflection between 2″, 3″ is very nearly straight and parallel to the ray (3).

The wave-fronts are obtained from the incident wave-front Ap ; the initial refracted wave-front, B_1qr (for which $\mu \,.\, Ar = pB_1$, &c.) and the initial reflected wave-front, $p_1q_1r_1$ (for which $p_1B_1' = B_1'B_1$, $r_1A' = A'r$, &c.).

By the construction it is demonstrated that the caustics, which are the loci of the meridional principal focal points, are also the loci of the cusps on the wave-fronts. Also that the reflected ray (3)—the chief ray of the reflected parallel pencil—is the locus of a point of inflection on the wave-front (§ 4 (II.)).

§ 6. *Relation between the Portions of the Line of Incidence Intercepted by the Refracting Sphere of Radius r and one of Radius μr.*

(I.) Let l or $2y'$ be the length of the portion of the ray (like BB′ in Fig. 2) between two points of incidence. Then (§ 2 (III.)).

$$YZ/\beta Z = f'/y' \text{ and } 1 + y/\beta Z = f'/y'.$$

For the pth point of reflection, let u_p, v_p be the respective distances from the point of reflection of the vertices of the incident and reflected pencils. Then (§ 3 (V.))

$$1/u_p + 1/v_p = 4/l.$$

For the $(p+1)$th point of reflection, $u_{p+1} = l - v_p$.

(II.) Write $\beta Z = n \,.\, Y\beta$. Consider successive reflections. Then $1 + 1/n = 2f'/l$. $\therefore\ f'/l = (n+1)/2n$.

$$\therefore\ u_1 = l - f' = l(n-1)/2n.$$

$$\therefore\ \frac{1}{u_1} = \frac{2}{l}\left(1 + \frac{1}{n-1}\right); \quad \therefore\ \frac{1}{v_1} = \frac{4}{l} - \frac{1}{u_1} = \frac{2}{l}\left(1 - \frac{1}{n-1}\right)$$

$$\therefore\ \frac{1}{u_2} = \frac{1}{l - v_1} = \frac{2}{l}\left(1 + \frac{1}{n-3}\right); \quad \therefore\ \frac{1}{v_2} = \frac{2}{l}\left(1 - \frac{1}{n-3}\right)$$

$$\frac{1}{u_p} = \frac{2}{l}\left(1 + \frac{1}{n-2p+1}\right); \quad \therefore\ \frac{1}{v_p} = \frac{2}{l}\left(1 - \frac{1}{n-2p+1}\right).$$

(III.) When the emergent pencil is a parallel pencil after p internal reflections, $l-v_p=f=f'$.

$$\therefore \quad v_p=u_1 \text{ and } n-1=-(n-2p+1). \quad \therefore \quad n=p.$$

When $n=2p+1$, $1/u_p=3/l$ and $1/v_p=1/l$.

Thus the image point is on the refracting surface.

When $n=2p$, $1/u_p=4/l$ and $1/v_p=0$. Thus the reflected pencil is one of parallel rays.

§ 7. *Conditions for a Rainbow.*

In order that a rainbow may be formed, the emergent, as well as the incident, portions of the effective pencil may be assumed to consist of parallel rays. Hence the image-points F′, I′, I″, &c., will be symmetrically distributed. Only certain pencils of incident rays will be so placed that their emergent portions consist of parallel rays.

When there is one internal reflection the incident pencil is in such a position that the second principal focus of its meridian rays is on the spherical surface.

When there are two internal reflections the incident pencil is in such a position that the second principal focus of its meridian rays is the first principal focus for the reflected pencil. Hence a parallel pencil passes between the two points of reflection.

When (§ 6 (III.)) there are $2p+1$ internal reflections, the $(p+1)$th point of reflection is a focus of the pencil reflected at the pth point.

When there are $2p$ internal reflections, a parallel pencil passes between the pth and the $(p+1)$th points of reflection.

Also (§ 6 (III.)) when the number of internal reflections is

one, $\quad f'=2y' \text{ and } \beta Z=Y\beta,$

two, $\quad f'=3y'/2 \quad \beta Z=2\,.\,Y\beta,$

three, $\quad f'=4y'/3 \quad \beta Z=3\,.\,Y\beta.$

And for n internal reflections, $\beta Z=n\,.\,Y\beta$.

The chief rays of these pencils are at minimum deviation (§ 9).

§ 8. *Construction for Finding the Point of Incidence of the Chief Ray of a Parallel Pencil that shall Emerge as a Parallel Pencil after n Internal Reflections.*

Let (Fig. 5) circles of radii r and $r\mu$ be drawn from a centre, C. Through C draw N′MC, cutting the former circle at M. Draw NMQ perpendicular at M to N′MC, cutting the circle of radius $r\mu$ at N. Make N′M=NM.

When there are n internal reflections, mark off PM equal to n

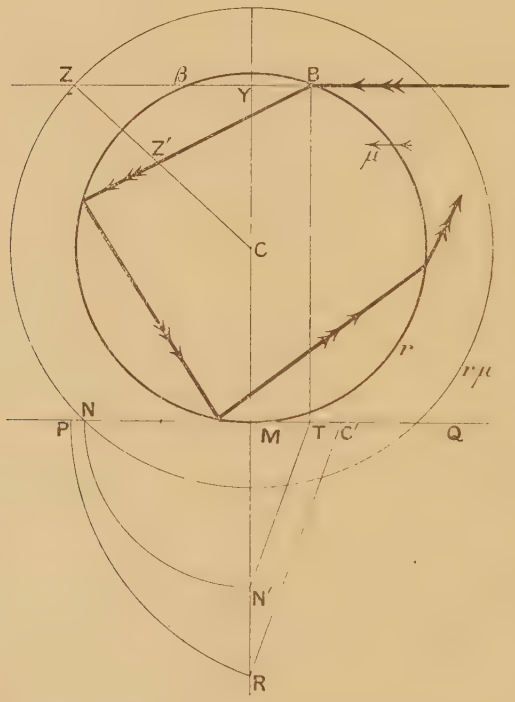


Fig. 5.—Construction for Point of Incidence of a Rainbow Ray.

units of length and MC′ equal to one unit. From C′ as centre describe a circle cutting MN′ at R.

Join RC′. Draw N′T parallel to RC′. Draw TB parallel to MCY, intersecting the circle of radius r at B. B is the required point of incidence of a ray, YB, incident at right angles to CY.

Let BY intersect the circles at β, Z.

Now $$\frac{MT}{NM}=\frac{MC'}{MR}=\frac{1}{\sqrt{n(n+2)}}.$$

Also $$NM^2=Z\beta \,.\, ZB=ZY^2-YB^2.$$

$$\therefore \quad ZY^2-YB^2=n(n+2)\,.\,MT^2.$$

Also MT, YB, BY are all equal.

$$\therefore \quad ZY^2/\beta Y^2=(n+1)^2.$$

$$\therefore \quad Z\beta=n\,.\,\beta Y.$$

§ 9. *Deviation. Minimum Values.*

(I.) Consider the light that emerges from the spherical body after one internal reflection. The semi-deviation of any ray is (*see* Fig. 4) the angle between CB_1 and the line joining C with the point of reflection. Then the chief ray (5), whose meridional principal focus is on the spherical surface is at minimum deviation. For the caustic from 4′ to 6′ is the locus of the intersection of adjacent refracted rays between (4) and (6), and these rays gradually diminish in slope. Hence all the refracted rays except (5) strike the spherical surface at points lower than 5′.

(II.) Consider the light that emerges from the spherical body after two internal reflections. The semi-deviation of any ray is the angle between CB_1 and the perpendicular from C on to the reflected ray. Then the chief ray (3), whose reflected pencil is a parallel one, is at minimum deviation. For the caustic from 2″ to 3″ (at infinity) and 3″ to 4″ is the locus of the points of intersection of consecutive reflected rays from (2) to (3) and from (3) to (4) respectively. Thus, for a ray between (2) and (3) there is a parallel one between (3) and (4) (§ 4 (II.)), and the semi-deviation of (2) is greater than that of (3).

(III.) The same chief-ray remains at minimum deviation when the associated pencil is converging or diverging.

ABSTRACT.

A method of drawing the cusped wave-fronts produced by refraction and reflection at a spherical surface and a graphic treatment of the elementary theory of the rainbow are described in the Paper. The method is based upon the properties of the centre of junction. In the case of reflection at a spherical surface in which aplanatic points are not available the position of the junction-centre is obtained by elementary geometry.

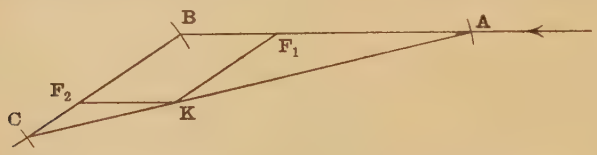
By the use of junction-centres the distribution of the successive foci obtained on refraction and reflection at a spherical surface is readily plotted. If in the case of a rainbow it is assumed that the emergent portion of the effective pencil, as well as the incident portion, is one of parallel rays, then the distribution of foci with regard to the drop is a symmetrical one.

The caustics on refraction and reflection are also readily drawn as loci of points and envelopes of rays. Hence the wave-surfaces can be obtained. These in some regions are cusped and the caustics are the loci of the cusps.

In the case of pencils that are effective in producing rainbows, the lengths of the incident portion of the chief ray intercepted by the spherical drop of radius, r, and a concentric sphere of radius, $r \cdot \mu$, are in the ratio 1 to $n+1$, where n is the number of internal reflections. This leads to a geometrical construction for finding the points of incidence of the chief rays of the effective pencils. These rays are at minimum deviation, and the same chief ray remains at minimum deviation although the associated pencil may be converging or diverging.

DISCUSSION.

Mr. T. H. Blakesley (communicated remarks) said: In Mr. Bower's treatment of the geometrical disposition of the optical foci in the case of a refracting and internally reflecting sphere, commonly, for brevity, described as the rainbow problem, he has certainly shown an unflinching courage in loading a single figure with an immense amount of accurate matter. He is without a doubt a stickler for geometrical treatment. But when an analytical form expresses a geometrical truth it must have taken that form under strict geometrical treatment, and what is called by mathematicians analytical treatment is really only the combination of analytical forms already established by geometrical treatment. In dealing with the very valuable theorem of M. Cornu, Mr. Bower does not make it quite clear that the coordinates of the junction point, measured respectively along the two directions of the bent ray, from the point of bending as origin, are indeed equal to the two focal lengths for the light in the two compartments formed by the boundary, in the case of refraction; in the case of reflexion the simpler



result of equality takes place. If ABC represents a ray undergoing refraction at B, and BF_1, BF_2 are the focal lengths in the two media, the junction point K will be found by simply drawing F_1K, F_2K parallel to BC, BA respectively, and if any small object is placed at A its image will be at C, in AK produced when necessary to cut BC in C.

For $$\frac{CF_2}{BF_2} = \frac{BF_1}{AF_1}.$$

Therefore, if $AF_1 = m \cdot BF_1$, it follows that $CF_2 = \frac{1}{m} \cdot BF_2$. But m or $\frac{AF_1}{BF_1}$ is the relation in linear magnification of the object to its image, and, therefore, the relation of the image to the object is $1/m$ or CF_2/BF_2—that is to say, C must be the position of the image. Viewed in this way M. Cornu's

theorem appears as a corollary of the fundamental law of conjugate foci. A word of further explanation should be added touching the *directions* of the elemental lines in the object and image, the comparison of which constitutes the relation m, or magnification ratio. In an axial pencil the incidence of the central ray is vertical, and the elemental lines are always considered at right angles to the axis. It would be more strict to describe them as parallel to the surface, for then the same description can be extended to the oblique cases. In the above figure if meridional rays are being considered the elemental lines of the object and image at A and C respectively are to be considered as parallel to the section of the surface at B, made by the plane ABC. For the rays which Mr. Bower describes as sagittal the elemental lines must be considered as parallel to the surface in a direction at right angles to the above. In dealing with the actual phenomenon of the ordinary rainbow it is not necessary, as Descartes has done, to introduce the idea of minimum deviation, and the consequent concentration of the light. That the colours are seen at all and in differing directions is sufficient to prove that the light is in a condition of sensible parallelism in its passage from the raindrop to the eye, and as it certainly is so before impact upon the drop the whole optical effect is that of a telescope in which the light is reflected between the object glass and eye-piece, as in Newton's telescope, it may be more than once. As the eye-piece has the same focal length as the object glass, the magnifying power is (-1). If there are two internal reflections there is an erecting day-glass arrangement not increasing or diminishing the magnifying power. But to be telescopic at all involves the necessity of the coincidence of two principal foci within the apparatus, and this consideration, together with that of symmetry, necessitates what Mr. Bower has pointed out, viz., the formation of a focus on the surface in the primary rainbow, and of parallel passage between the two reflections in the secondary rainbow. Abnormal rainbows are sometimes to be seen. By this expression I mean bows in which the axis does not coincide with the straight line joining the sun with the anthelial point. They are not readily explained, but are probably due to portions of ice or air in water, and the direction of gravity seems to be a factor in this formation. But, however they are constituted, the telescopic fact stated must be true of them, there must be coincidence of principal foci. Mr. Bower's methods are admirably adapted for all direct problems upon the subject, such as the impossibility of a primary rainbow for an index in excess of 2, and generally for the examination of cases where the index is taken as known; but should the problem be to find the index for which the primary bow coincides with the secondary, it is unlikely that pure geometry could afford the answer—viz., 1·31201248.

Mr. W. R. Bower (communicated reply) writes: A graphic method cannot displace analysis when a computation is required to a high degree of accuracy. But experience is necessary when symbolical forms have to be interpreted, and, therefore, an analytical treatment of geometrical truths often hides for a time facts that should be exhibited. Educationally, graphic methods, that are not purely empirical, are more enlightening than analytical. The rainbow problem gives an example. In the usual algebraic discussion the parallelism of the emergent light is a condition in the proof of minimum deviation. We infer from the geometrical treatment—as mentioned at the end of the Paper—that the ray at minimum deviation may be the chief ray of pencils that are not necessarily parallel. In Mr. Blakesley's construction for the junction-centre, it is assumed that the positions of the principal foci are known. In the Paper in Vol. XXV. (p. 172) of the "Proceedings" the reverse process of obtaining the focal points from the junction centre is given. The junction centre is deduced and the discussion based entirely on the index of refraction, the centre and radius of curvature and the assumption that the radiant point travels along a fixed incident ray. No use is made of the sine law of refraction. The procedure follows a course nearly similar to that which would be taken if the magnification relation was assumed.

XXV. *On Gyrostatic Devices for the Control of Moving Bodies.*

By JAMES G. GRAY, *D.Sc., F.R.S.E., Lecturer on Physics in the University of Glasgow.*

RECEIVED MAY 4, 1914.

IT is the object of the present Paper to describe a number of gyrostatic devices available for the control of moving bodies, such as torpedoes, submarine craft, airships and aeroplanes. These contrivances have suggested as a kind of by-product a variety of gyrostatic bicycles and motor-cars, both two-wheeled and four-wheeled. The stability of the gyrostatic system is, however, in all the cases considered in the present Paper, derived directly or indirectly from the propelling system. Hence these cases do not include solutions of the monorail problem; for they have not true stability when they are at rest or moving in the backward direction. Further, it will be seen that the tandem-wheeled motor-cars to be described, although they may be set to run in a perfectly straight path, will not balance on a single rail. The devices, however, have properties which are not possessed by any of the monorail devices so far evolved, properties which may possibly render them of the greatest value to a nation whose continued existence depends on its ability to retain supremacy on sea, and to obtain the supremacy of the air.

With a view to making clear the action of the devices I shall first show some old experiments and will make some deductions which are not obvious. In Fig. 1 is shown a gyrostat set up in a fork and pedestal mounting. The arrangement is substantially that made use of long ago by Léon Foucault in demonstrating the earth's rotation. The flywheel possesses three freedoms. The fork is capable of rotation, with but little friction, about a vertical axis, and the gyrostat is carried on knife-edges formed in the prongs of the fork, and is thus free to turn in the line joining the knife-edges. The axis of rotation of the flywheel XX′ and the two axes just specified—namely, ZZ′ and YY′—are mutually perpendicular when the axis of the flywheel is horizontal. Under these conditions the instrument is, of course, freely mounted.

The curved rod terminating in an arrow-head shows the direction of spin. The straight rods are intended to represent the angular momentum of the flywheel and the applied couple respectively. With the direction of rotation shown, the

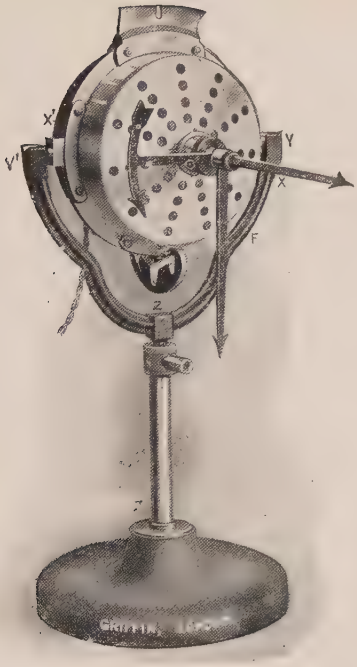


FIG. 1.—GYROSTAT IN "FORK AND PEDESTAL" MOUNTING.

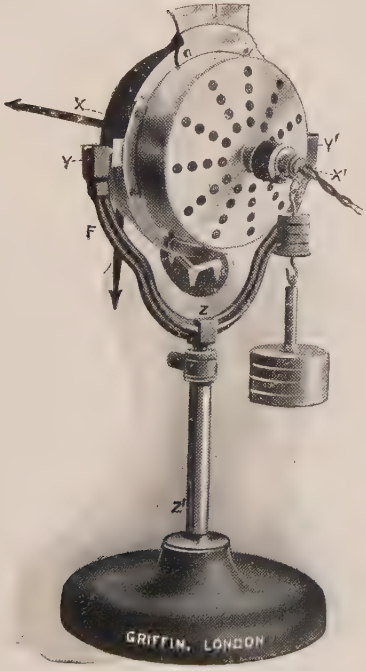


FIG. 2.—GYROSTAT IN STEADY PRECESSION, WITH ITS AXIS HORIZONTAL.

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angular momentum or spin is, according to the usual convention, represented completely by a straight line of proper length, drawn at right angles to the plane of the flywheel, as in the figure. The second straight rod can be moved round in a plane parallel to that of the flywheel ; it is held in any position by means of a spring washer. These lines, which represent the " spin " and the applied couple, are called the spin-axis and couple-axis respectively.

The model is available for demonstrating the principal properties of the gyrostat. By its means it is easy to show that if a couple is applied to the gyrostat, so as to cause precessional motion, the motion is such that the spin-axis moves towards the instantaneous position of the couple-axis. This rule always holds. A couple tending to turn the gyrostat about the fork-axis causes precession about the pedestal axis; one tending to cause turning about the pedestal axis causes precession about the fork axis.

In Fig. 2 the gyrostat is shown in steady precession, with the axis of its flywheel horizontal, under the action of a suspended weight. If W is the weight, l the distance of its line of action from the axis of the knife-edges, I the moment of inertia of the flywheel, and ω_1 its angular velocity, the gyrostat and suspended weight turn about the vertical axis with angular velocity ω_2, where

$$Wl = I\omega_1\omega_2.$$

Providing that the vertical axis is frictionless and the spin is maintained constant the weight will be carried round at a constant rate ω_2. In practice, of course, this condition is not fulfilled ; there is in general a small oscillation about the steady motion, and, in consequence of friction, there is a gradual descent of the weight with slow variation of ω_2, which (with constant spin) will be an increase or a diminution according as the couple Wl is increased or diminished by the descent. There is a loss of energy on the whole. If the weight descends through a distance d, its loss of potential energy is Wd, and, since for rapid spin ω_2 is small, it is substantially this potential energy that is converted into heat by friction at the vertical axis.

The work done in a given time, T, is $L_1\omega_2 T$, where L_1 is the frictional couple. Now the greater the value of ω_1 the smaller is ω_2, and the less is the work done against friction in a given time.

Again, suppose that with the gyrostat in steady precession, with its axis horizontal, a couple of moment L_2 is applied in a horizontal plane in the direction of the precessional motion.

If L_2 is greater than L_1 the effect will be to raise the weight against gravity. The work done by this couple on the weight in time T is $(L_2—L_1)\omega_2 T$, and hence the rate of working is proportional to ω_2. This is of great importance from the point of view of the application of the gyrostat as a transmitter of work and as a stabilising agent. For a given value of ω_1, ω_2 is proportional to W, and hence the greater W, the greater is the amount of work done by the net couple $L_2—L_1$ in a given time. If $L_2—L_1$ is small and W great the action is the analogue of the ascent of a heavy weight along an inclined plane of very small slope under the action of a force applied up the plane. Again it will be seen that by making ω_1 very large—that is, by rotating the flywheel at a very high speed—a large weight may be raised through a sensible distance without $\omega_2 T$ being necessarily great, if $L_2 - L_1$ be made large enough. This point is of importance.

Now let the side weight be removed, and a top weight be added to the frame of the gyrostat, so as to bring the centre of gravity of the system vertically above the line of the fork bearing when the axis of the flywheel is horizontal. When so arranged the gyrostat is unstably mounted in the fork, and if the flywheel is spinning any tendency to tilt will result in precessional motion ; tilting to one side causes precessional motion in one direction, tilting to the opposite side reverses the direction of the precessional motion. Further, if the gyrostat is precessing in one or other direction and a couple is applied to the fork in the direction of the precessional motion the gyrostat tilts on the fork bearing, so as to raise the centre of gravity of the arrangement—that is, so as to annul the tilting couple.

It should be observed that here the gyrostat acts as a transmitter. The couple applied to the fork (which may be called the stabilising couple) is applied in a horizontal plane, the gyrostat and its attachments turn instantaneously in a vertical plane. The precessional motion about the pedestal axis ceases when the centre of gravity of the system is vertically above the line of the fork bearings. If the stabilising couple goes out of existence at the instant at which the precessional motion ceases, and the motion about the horizontal arm be, as it usually is, very slow, the gyrostat is practically left in the upright position.

Precessional motion about the pedestal axis in one or other direction follows on tilting of the gyrostat on the fork bearings. If now a mechanism is devised whereby the fork is for-

cibly turned in the direction of the precessional motion immediately such precessional motion takes place, and further if the stabilising action ceases at the instant at which the necessity for its existence disappears, it is clear that the gyrostat will be maintained in the upright position.

In Fig. 3 is shown a form of stilt top devised by Prof. H. A. Wilson, and exhibited by him to the Physical Society in 1907. A gyrostat is mounted as shown in a frame, *f*. The gyrostat frame is on cross bearings carried by *f*. When *f* is upright these bearings are in a vertical line. The crank *c*, which is rigidly fixed to the frame of the gyrostat, is attached to one end of a stretched spring *s*, the other end of which is fastened to a point, *p*, in the main frame. If the flywheel of the gyrostat is set spinning and the top placed on a table with the plane of the flywheel and the main frame, *f*, in the same vertical plane, and left to itself, it will balance for a considerable time if the spin is great. Initially *f* is in the same plane as the spring *s*.

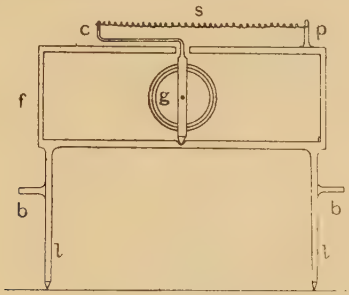

FIG. 3.—STILT TOP.

The stretching force in the latter, therefore, exerts no moment on the gyrostat about the cross bearings which attach it to *f*; but as soon as the gyrostat precesses on the latter bearings the crank gets out of line with the frame, and the spring exerts a moment in the direction of the precessional motion.

The entire top, when vertical, is unstable, without rotation of the gyrostat flywheel, about the line of contact of the feet with the table. Further, in consequence of the stretched spring, the gyrostat is unstably mounted on the frame. Thus the gyrostat is doubly unstable without rotation of its flywheel.

Starting with *f* in a vertical plane containing the crank and spring we may suppose it to tilt over on the table. As a consequence the gyrostat precesses about the cross bearings and the

precession is aided by the spring, with the result that the frame erects itself into the vertical. But at the instant at which the frame has attained the vertical the spring is out of line with the frame, and is exerting a moment on the gyrostat. Under the influence of this couple the gyrostat continues to precess about the line of contact of the feet with the table ; that is, the main frame passes beyond the vertical position, after which the lateral instability of the entire structure results in the establishment of a couple tending to accelerate this precessional motion. This couple causes precession about the cross-bearing bringing the crank and spring into line with *f*, but when this alignment occurs, the entire top is inclined from the vertical; and so on. The amplitudes of these oscillations continually increase, and finally the top falls over.

Again, suppose that, starting as before with *f* and the crank in one vertical plane, the crank gets out of line with *f*. As a result the spring exerts a moment on the gyrostat, which, in consequence, precesses about the line of contact of the feet of the top with the table. This precessional motion is automatically accelerated and the spring is thrown into line with *f*, which is now inclined from the vertical, and so on.

It will thus be seen that starting with the main frame and the spring contained in one vertical plane the top balances ; and if the spin is great the balancing power is very considerable. But there is not true stability. The frame oscillates to and fro on the legs, the gyrostat oscillates to and fro on the bearings which carry it in the frame. If the stability were real the top, if started in an inclined position, would erect itself into the vertical one with the spring in the plane of *f*.

It is interesting to consider this matter from the energy point of view. The entire structure is unstable on the legs, and thus possesses a stock of potential energy. Again, potential energy is stored in the spring. When the frame tilts on the legs and the gyrostat turns on the frame bearings energy is dissipated in friction. Consequently once the frame has become inclined to the vertical, or the crank has got out of line with the frame, the system cannot return of itself to the position of maximum potential energy, that is, to the position in which the frame and crank are in one vertical plane.

In Figs. 4 and 5 are shown two further experiments in which a gyrostat is mounted in such a manner as to possess two instabilities without rotation of its flywheel. In Fig. 4 is shown a new form of stilt top designed by the writer. A gyrostat is

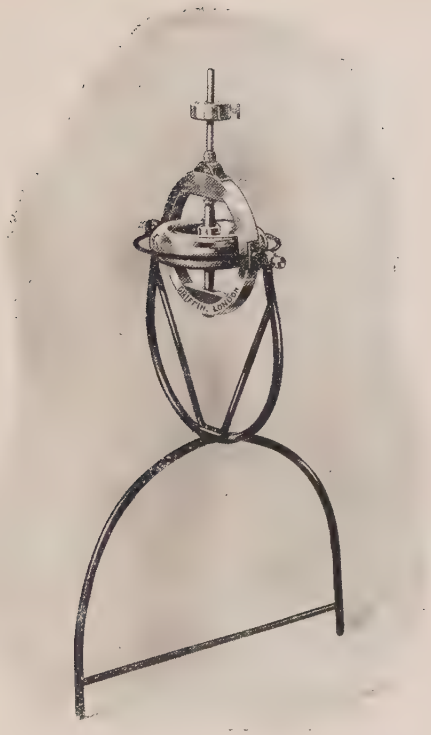

FIG. 4.—FORM OF STILT TOP.

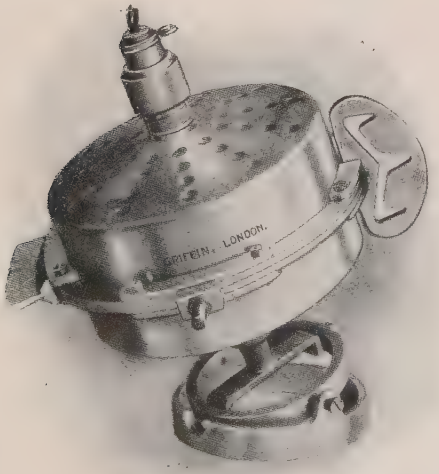

FIG. 5.—GYROSTAT ON GIMBALS.

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carried, with its axis vertical, on two horizontal bearings arranged in a frame terminating in two legs as shown. When the supporting legs and the axis of the gyrostat are vertical, with the weight in the upper position, as shown in the figure, the gyrostat is doubly unstable without rotation. If the flywheel is spun rapidly and the top set up as described, it displays considerable balancing power. But from what has been said it will be evident that this arrangement does not provide an example of true gyrostatic stability. Oscillations about the line of contact of the feet with the ground and about the horizontal bearings quickly grow up.

The top is well adapted to show the necessity for two instabilities without rotation of the flywheel. If the arrangement is set up as shown, but with the weight vertically below the gyrostat, the top exhibits no balancing power.

Fig. 5 shows Lord Kelvin's "gyrostat on gimbals" experiment. The gyrostat is supported on a universal joint, or pair of gimbal rings. When the gyrostat is in the vertical position it clearly possesses two instabilities without rotation, one about each gimbal axis. In performing the experiment the flywheel is rotated rapidly and the arrangement placed on the gimbal rings, with the axis of the gyrostat vertical, and left to itself, when it balances on the ring in contact with the table. The upright position is soon departed from, and is never regained. The axis moves round its initial position, its distance from the latter continually increasing. Energy is continually dissipated at the gimbal axes, which energy is derived from the potential energy possessed by the arrangement by virtue of the peculiar manner in which it is mounted.

Returning now to the top shown in Fig. 3 it will be seen that I have added to Mr. Wilson's arrangement the two projecting pieces, *bb*. So designed it may be set up with these projections engaging on the knife edges arranged in the fork of the apparatus shown in Fig. 1. The arrangement is shown in Fig. 6. The flywheel of the gyrostat is set into rapid rotation and the arrangement mounted on the fork with the frame *f* and the crank in one vertical plane. The fork is grasped in the hand of the experimenter. Now suppose the arrangement to tilt on the fork bearings b_1, b_2. The gyrostat precesses on the bearings that carry it in the frame, and immediately a couple, due to the spring, tending to accelerate the precessional motion, comes into existence. At the same time, the experimenter turns the fork so as to bring the frame into line with the crank.

Providing this operation is properly carried out, the frame is restored to the upright position and the crank is in line with it. The spring has supplied energy to the frame in restoring it to the vertical position, the potential energy lost by the spring has been made good by the experimenter.

Now let a weight, w, be attached to one side of the frame. This at once causes precession of the gyrostat and the establishment of a couple due to the spring. The experimenter turns the fork so as to bring the frame into line with the crank. Here energy is being transmitted from the spring to the frame by means of the gyrostat, and at the same time energy is being supplied to the spring by the experimenter. The frame turns on the fork bearings so as to raise the weight against gravity. The precessional motion continues until the centre of gravity of the entire arrangement is vertically above the line of b_1 b_2.

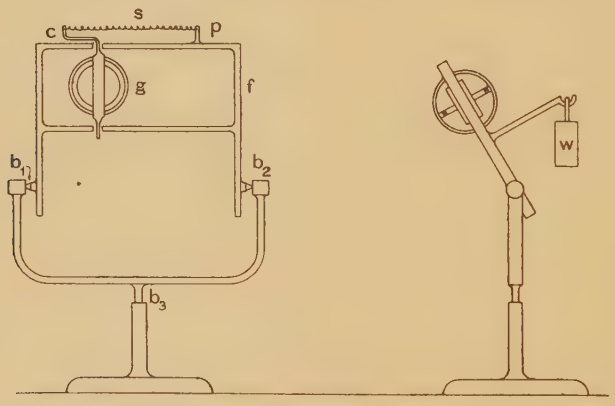

Fig. 6.—Stilt Top set up on Fork and Pedestal Mounting.

The spring is now in line with the frame, and consequently its stock of potential energy is precisely that which it possessed at the start of the experiment. The energy required to raise the top against gravity has been supplied by the experimenter.

The action of the top shown in Fig. 7 is identical with that of the one just described. The crank and spring are placed transverse to, instead of in the plane of, the main frame.

Fig. 8 illustrates the applications of the principles just described to the construction of two-wheeled and four-wheeled gyrostatic motor cars. The figure shows a car in which the wheels, of which there are two, run in tandem. The gyrostat g is mounted on top and bottom bearings provided in the main

frame F. One of the axles which carry the gyrostat is extended and terminates in a bearing for one of the wheels w_1 of the car. The construction is such that this wheel is in the plane of the flywheel of the gryostat. The back wheel of the car is geared up to driving mechanism. The gyrostat is fitted with a crank

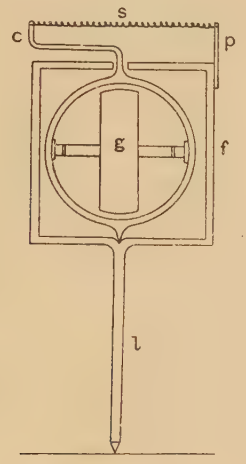

Fig. 7.—Stilt Top.

and spring device as already described. The wheel w_1 and the flywheel of the gyrostat are in the same plane. Arrows on the wheels indicate the direction of motion of the device.

Let the flywheel be set into rapid rotation and the car

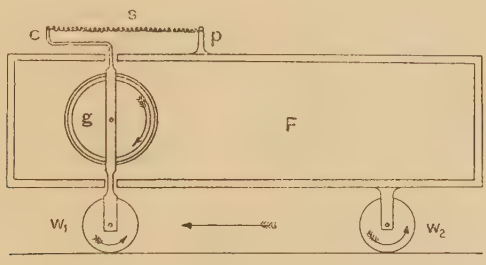

Fig. 8.—Two-Wheeled Motor Car.

placed on the floor with the main frame, the flywheel of the gyrostat, the two wheels supporting the device and the crank in one vertical plane. It will be clear from what has been said that, if left to itself, the car will balance on the wheels, but with

the accompaniment of gyrostatic oscillations. If it were allowed to remain stationary the device would eventually fall over; but when driven in the forward direction it is completely stable. This stability is obtained in the following way: The gyrostat steers the car, and when precessional motion on the cross bearings takes place this results in the main frame F, to which the end of the spring remote from the crank is attached, being brought into line with the crank. Thus the action is precisely that described above for the stilt top when mounted in the fork bearing. The stability is complete when the car is moving in the forward direction. If a weight is attached to one side the car banks up against the weight so as to bring the centre of gravity directly above the wheels, and thereafter proceeds in a straight line path.

In this device the gyrostat derives the stabilising forces from the spring, and when the stock of potential energy possessed by the spring is drawn upon an equal amount of energy is automatically supplied to it by the propelling system. The gyrostat thus detects a tendency to tilt in either direction, calls upon the spring to supply the necessary correcting forces, and upon the propeller to maintain constant the energy possessed by the spring.

The gyrostatic action of this device is illustrated graphically in Fig. 9. We suppose the car to start perfectly balanced and upright. This condition is shown in (1) of the upper diagram (A) of the figure. The arrow at the back of the car shows the direction of motion, and the curved arrow attached to the gyrostat indicates the direction of rotation of the flywheel. The angular momentum may be completely represented by a straight line drawn out from the flywheel towards the reader. Thus, a_1 is the spin-axis. Now, the car is unstable about the line of contact of the wheels with the table, and hence, after a short interval, a tendency to tilt in one or other direction will assert itself. We suppose that when the car is in the position (2) there exists a tendency of the device to tilt towards the reader. This tilting couple is completely represented, according to the usual convention, by a line a_2 (the couple axis), of proper length drawn as shown in (2) towards the back of the car. The gyrostat precesses, so that a_1 turns towards the instantaneous position of a_2, the crank comes out towards the observer, and a couple tending to turn the gyrostat counter-clockwise, as viewed from above, is established. This couple is represented by a_3 in (3) above. The gyrostat now precesses

so that a_1 moves towards the instantaneous position of a_3—that is, the car erects itself against gravity; it moves away

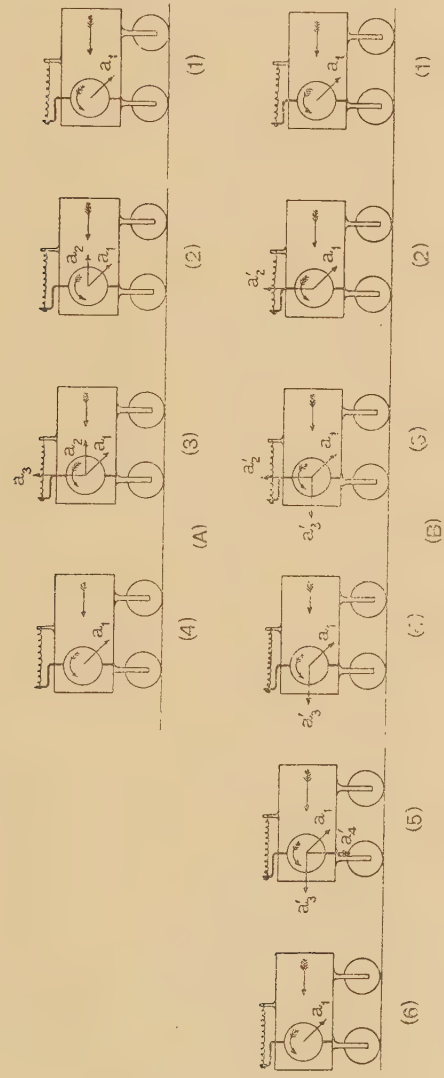

FIG. 9.—ACTION OF TWO-WHEELED GYROSTATIC MOTOR CAR.
A.—March of Vectors following on Tilting of Car towards Reader.
B.—March of Vectors following on Displacement of Spring from Central Position.

from the reader about the line of contact of the wheels with the ground. The tilting couple is thus reduced and finally an-

nulled. Further, in consequence of the fact that the gyrostat is steering the car, which is moving in the forward direction, the want of alignment of the crank and car is being reduced. Thus, a_2 and a_3 tend to diminish together, and the car is finally left as in (4) of the top diagram of the figure.

We may now investigate the annulment of the instability introduced by the crank and spring. This will be understood from the lower diagram (B) of the figure. In (1) the car is shown balanced in the upright position with the crank in line with F. We suppose it to get out of line with the car without there being any tendency of the latter to tilt (this may be due to a jolt). Let the displacement of the crank be towards the reader. The couple brought into existence by the want of alignment is represented by a_2' of (2), and the gyrostat turns so that a_1 moves towards the instantaneous position of a_2'; the car turns over from the upright position (away from the reader), and a couple represented by a_3' of (3) comes into play. The gyrostat now turns on the vertical bearings in the direction which results in a_1 moving towards the instantaneous position of a_3'—that is, in the direction which brings the crank into line with the car. The couple due to want of alignment of the crank and frame disappears, but the car is left inclined to the vertical. This stage is shown in (4). The gyrostat is now precessing on the vertical bearings, and a couple represented by a_4' (5) is introduced. The resulting precessional motion and the forward motion result in a_3' and a_4' going out of existence, as already explained, and the device is left as shown in (6). It is now upright, with the crank and frame F in line.

As we have already seen, the effect of attaching a weight to one side of F is to cause the car to bank up and then to pursue a straight path. The properties of the device are, however, greatly added to by attaching a weight, w, to the frame of the gyrostat, as shown in Fig. 10 (a) and (b). If the gyrostat is spinning in the direction in which the wheels of the car rotate, the weight should be placed as in (a); if the direction is reversed the weight should be as in (b). Consider diagram (a). Let a side weight (Fig. 6) be supposed attached. The car banks up against this weight, with the result that the line of the frame bearings becomes inclined to the vertical. A couple acting against the spring is then applied by. A steady state is now arrived at, in which the couples applied to the gyrostat by the spring and by the weight are equal. The car is not sufficiently banked

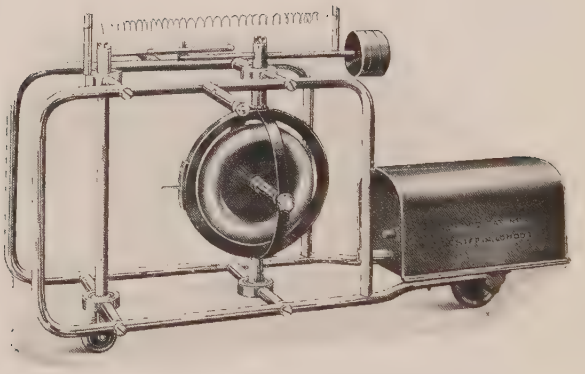

Fig. 11.—Gyrostatic Motor Car.

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up to account entirely for the side weight, and the gyrostat precesses continually. The car moves in a circular path.

When the side weight is removed the device straightens itself out and proceeds in a straight path. Attaching the side weight to the other side of the car causes the latter to move in a circular path in the opposite direction to the former one.

Fig. 11 is a photograph of an actual working model of a two-wheeled car constructed on the above principles. The front wheel is the driven one. The gyrostat is carried on the main frame on vertical bearings. It steers the back wheel through a link attachment. The car is available for demonstrating the action of a monorail car, or the gyrostat can be placed under the control of a weight as just described. It can then be set to move in a curved path in either direction. A much larger model, provided with an electromagnetic steering device

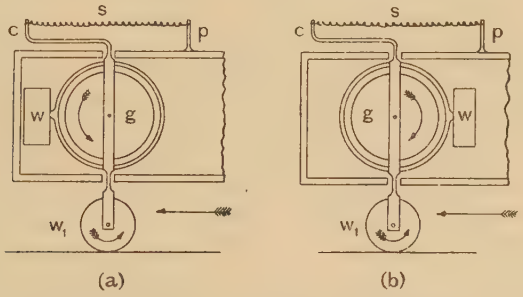

FIG. 10.—GYROSTATIC MOTOR CAR FITTED WITH CONTROLLING WEIGHT.

capable of being actuated by the wireless transmission of electrical action, is in course of construction.

Fig. 12 shows the application of the principle to the steering of a torpedo or dirigible airship. The upper diagram shows a side elevation, and the lower a plan of the arrangement. The gyrostat is mounted, with its axis horizontal, on cross bearings, b_1 b_2, carried by a frame, f. This frame is mounted on fore-and-aft bearings, b_3 b_4, attached to the moving body, and the construction is such that the system composed of gyrostat and attachments is laterally unstable on the body. The spring s_1 and the crank c render the gyrostat unstable relatively to the frame f. The moving body is supposed to be stable of itself. It will thus be seen that the gyrostat possesses two instabilities without rotation of its flywheel. By causing the gyrostat to

steer the body these two instabilities without rotation result in complete stability when the flywheel is rotating and the body moving in the forward direction.

In the construction of wheeled vehicles it has been found sufficient to connect up the gyrostat directly to the steering wheel or wheels. In the case where a steering mechanism, such as a rudder or plane, has to be operated forcibly this is not possible. Apparatus for operating a vertical rudder is shown in the lower diagram of Fig. 12. One end of a cord is attached to a point on the frame of the gyrostat. The cord is then passed once or more times round a vertical drum or pulley, d_1, and its free end attached to a point on a drum, D. A second cord is likewise attached at one end to a point on the opposite side of the frame of the gyrostat, passes once or more times round a second pulley or drum, d_2, and is attached to the

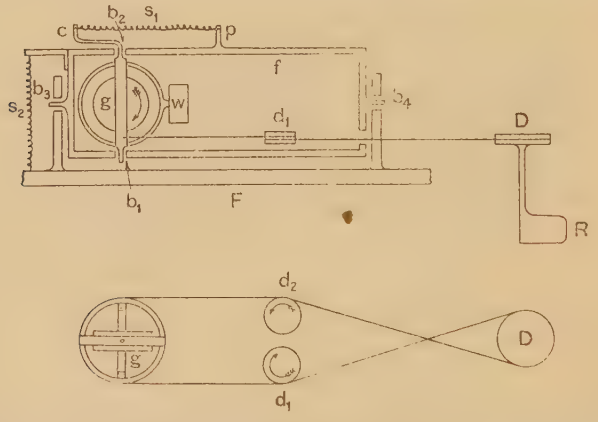

Fig. 12.—Dirigible Torpedo, with Stabilised Gyrostatic System.

opposite side of the drum D. The two pulleys d_1, d_2, which are of equal diameter, are geared up to a small electric motor; they revolve in opposite directions with the same speed. If the gyrostat precesses one of the cords (say c_1) attached to it becomes taut. A small stretching force in the cord on the gyrostat side of d_1 gives rise to a large stretching force in the cord on the drum side of d_1. If the stretching force on the gyrostat side of d_1 is zero that on the drum side is also zero. It will thus be seen that a small couple applied by the gyrostat results in the application of a very large couple to the drum, and hence to the rudder.

The rudder is so connected up that when the gyrostat precesses the body is steered up parallel to it, that is, so as to maintain the axis of the flywheel transverse to the body.

In the form of torpedo at present in use the gyrostat is freely mounted on gimbal rings. In the absence of a disturbing couple the axis of the gyrostat will retain its direction in space unaltered. Hence when the torpedo deviates in its path a shift occurs between the direction of the axis of the gyrostat and that of the projectile. The apparatus must be made with great precision; notably the centre of gravity of the gyrostat must coincide exactly with the point of intersection of the gimbal axes. If this condition is not precisely fulfilled the torpedo will travel in a curved path.

The existing type gives very good results over the short distances; but a gyrostat freely mounted would be useless in a long-distance projectile, even if a motor gyrostat were substituted for the one now employed. This point is not as a rule understood. In a dirigible torpedo, properly so called, the gyrostatic apparatus should be such that the gyrostat is endowed with complete stability. This condition fulfilled, the gyrostat can be caused to bring about turning movements of the torpedo by the application to it of tilting couples.

The device just described is well adapted for use on small dirigible airships. It is easy to contrive apparatus on hydrostatic principles which will cause an airship to ascend to a given height and to remain at that height.

Let, now, the apparatus be supposed mounted on an aeroplane, the bearings b_3 b_4 being fore and aft. The gyrostat is balanced on b_3 b_4 and a lateral tilt of the aeroplane would bring about a shift between it and the gyrostat, which might be utilised to operate the balancing apparatus. The gyrostat, it is to be observed, is maintained automatically in the position in which its axis is horizontal and across the aeroplane. Thus this gyrostatic device, as well as a further one to be described, would appear to be available as a detector and corrector of lateral tilting.

In the device as illustrated the axis of the gyrostat is across the moving body. It is possible to mount the gyrostat, doubly unstable as before, on the body with the axis of its flywheel fore and aft. The frame of the gyrostat is suitably connected up to the steering apparatus in such a manner that the moving body is maintained with its length parallel to the axis of the

flywheel. A gyrostat so mounted would appear to be available as a detector and corrector of longitudinal tilting.

In order that a gyrostatic aviator should confer both longitudinal and lateral stability upon an aeroplane it must be mounted with complete stability on the latter with its axis vertical. This would be easy on the principles explained, if it were feasible to steer the aeroplane in a vertical plane. This, of course, is not practicable.

Attention is now directed to Fig. 13, which shows a new form of stilt top. A gyrostat is pivoted within a structure terminating in two stiff legs. When the feet of the top are supported on a table with the plane of the frame vertical the line of the pivots which carry the gyrostat is sloped to the vertical,

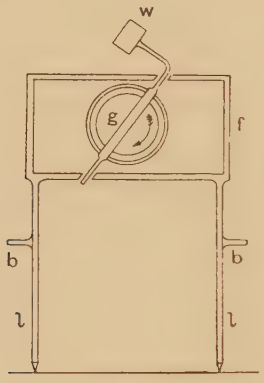

Fig. 13.—Stilt Top.

and with the direction of slope indicated it will be seen that when the plane of the flywheel coincides with that of the frame the weight w is constrained to move, relatively to f, in a circle whose highest point coincides with that occupied by w in the figure. Thus in the position shown the gyrostat, in consequence of the presence of the weight, is unstably mounted on the frame. Further, the frame is unstable about the line of contact of the feet with the table. Thus the gyrostat possesses two instabilities without rotation of its flywheel. If the flywheel is rotated rapidly in either direction and the top placed on a table as described, and left to itself, it will balance on the table. It will be readily seen, however, that the stability is not true stability; gyrostatic oscillations soon grow up.

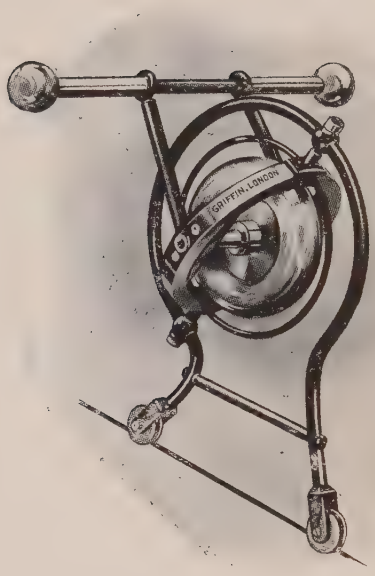

FIG. 14.—POLE-BALANCING TOP.

FIG. 15.—GYROSTATIC BICYCLE.

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Fig. 14 shows a pole-balancing top constructed on this principle. The frame terminates on two wheels adapted to run on a tight or slack wire. Attached to the gyrostat frame are two arms which support a pole weighted at both ends. This pole provides the necessary instability of the gyrostat relative to the frame. The flywheel is rotated clockwise as viewed by an observer from one side with the pole on his left, and the arrangement set up with the frame vertical and the pole horizontal. With this direction of spin the tilting of the frame to one side of the wire causes the pole to be carried over to the other side.

Returning now to the first of the tops just described, let it be spun and set up in the fork and pedestal mounting (after the manner of Fig. 6), with the frame and flywheel in the same vertical plane. As before, the experimenter operates the fork. With the direction of spin indicated in the figure tilting of the frame to one side of the fork causes the weight to be carried over to the other side. Now let a side weight w' be attached to the frame f. The gyrostat precesses on the frame bearings and w is carried (Fig. 13) over to the side of the frame remote from the attached weight; let the fork be turned by hand in the direction in which the gyrostat precesses, so that it follows up the latter. Providing that the fork is not turned too quickly the gyrostat will continue to turn on its bearings. It is to be observed that at any instant the acting tilting couple is the difference of the moments about the fork axis due to the side weight w' and w respectively. The effect of turning the fork is to diminish the moment due to w and the precessional motion is maintained. This action, it will now be shown, has resulted in apparatus of great beauty.

Fig. 15 illustrates a form of gyrostatic bicycle. A gyrostatic bicycle rider is mounted upon a safety-machine. The frame of the gyrostat is attached to the bicycle by means of a sleeved joint, and a pair of arms, carried by the frame of the gyrostat, are attached by means of pivots to the handle-bar of the machine. The construction is such that when the wheels are in one vertical plane the sleeved joint, referred to above, is considerably inclined to the vertical, and this results in the brass ball (*see* the figure) conferring considerable instability upon the gyrostat relatively to the bicycle. The entire machine is, of course, unstable on the wheels. These two instabilities, without rotation of the gyrostat flywheel, are available to annul one another with rotation.

The rider of a bicycle keeps his machine upright by operating

his handle-bar. When the machine tilts over to the left the rider instinctively turns the handle-bar to the left, and the forward momentum of the bicycle and rider aided by the gyrostatic action of the wheels (a relatively small factor), results in the erection of the machine. Similarly, if the machine tilts to the right the handle-bar is turned to the right.

The action of this gyrostatic bicycle is entirely different, and much more beautiful. The gyrostat is spun in the direction opposed to that in which the wheels of the cycle rotate when the latter is moving in the forward direction. Tilting of the machine to the right causes the gyrostat to precess, so that the brass ball and the front wheel and the brass ball move over to the left. As described above, energy is supplied from the forward motion, and the precessional motion ceases when the tilting couple is annulled. The stability, when the bicycle is moving in the forward direction, is complete. If brought to rest, following on motion in a straight path, it is left perfectly upright, and it then balances for a considerable time. Gyrostatic oscillations, however, grow up, but on the restarting of the machine these at once disappear. If the bicycle is started in an inclined position it erects itself into the upright position, and thereafter pursues a straight path.

Starting with the bicycle moving forward in the upright position, let a weight be placed on one side of the frame. The machine now proceeds in a circular path, the action being that described for my stilt top when set up in the pedestal mounting. Suppose the weight to be attached to the right-hand side of the frame as seen by an observer viewing the bicycle from behind. The front wheel and the brass ball are carried over to his left. The fact that the gyrostat steers the machine results in the latter turning continually, so as to annul the moment due to the brass weight; at the same time the precessional motion turns the ball so as to increase its tilting moment. A steady state is soon arrived at, and the bicycle moves in a circular path.

Fig. 16 is a diagrammatic representation of a large motor car constructed on the above principles. It will be seen that the gyrostat stabilises the entire structure and at the same time operates the steering wheel.

The function of the weight W_2 is to apply the necessary tilting couples. W_2 is carried on an arm which is rotated about a vertical axis by means of a small geared electric motor. When completed this car will be capable of being operated by wireless transmission of electrical action.

It should be noticed in the case of this bicycle and the motor car, that, provided the gyrostat is sufficiently powerful, the controlled device cannot possibly upset. The frame is continually following up the gyrostat, which thus cannot lose control.

Fig. 17 shows in side elevation a gyrostatic device adapted for steering a body stable of itself, such as a tricycle, four-wheeled motor-car or torpedo. The gyrostat is mounted on

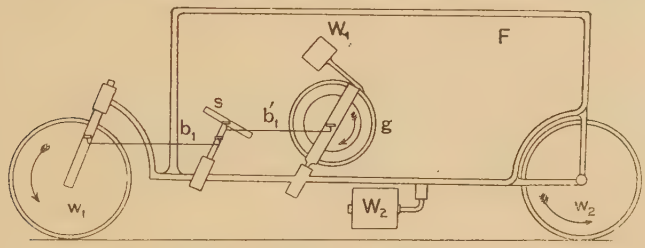

FIG. 16.—GYROSTATIC MOTOR WITH ELECTROMAGNETIC STEERING DEVICE.

bearings b_1b_1 carried by a frame, f; it is made azimuthally unstable by sloping the line of these bearings to the vertical, and attaching a weight, w, to the frame of the gyrostat. The frame is carried on horizontal bearings b_2b_2 arranged in pillars p_1p_1 attached to the moving body. The frame f is rendered laterally unstable on these latter bearings by attaching to it the

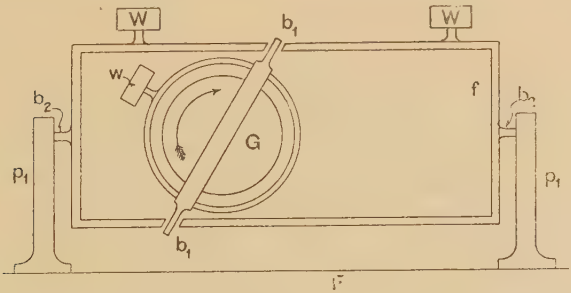

FIG. 17.—GYROSTATIC CONTROL FOR TORPEDO OR AIRSHIP.

weights WW, as shown. The gyrostat clearly possesses two instabilities without rotation of its flywheel. When the gyrostat is suitably connected up to the steering mechanism these two instabilities give rise to complete stability of the gyrostat when the body is in motion. The frame f may conveniently carry apparatus for applying tilting couples to the gyrostat.

In arriving at a conclusion as to the merits of the gyrostatic controls that have been described it should be remembered that the action of the gyrostatic system is a function of the angular momentum of the flywheel, of the double instability of the gyrostat, and of the forward momentum of the body. These can be varied to meet particular cases. In general, the greater the speed of the moving body the better is the behaviour of the device. The experiments exhibited show that the contrivances possess great power even when the speed of the controlled body is very small.

It may be pointed out in connection with the applications to the problem of the dirigible torpedo that by arranging that the instabilities of the gyrostat are small its properties are made those of a gyrostat freely mounted. It, however, remains correctly balanced on its mounting so long as the spin is main-

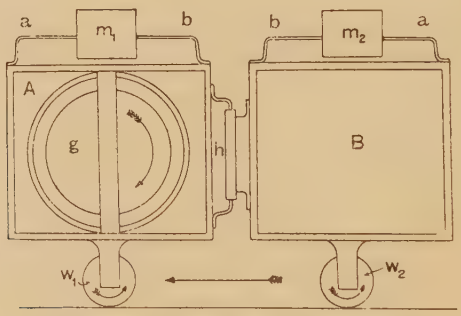

Fig. 18.—Gyrostatic Motor Car.

tained and the body is in motion. Further, if it is disturbed by any cause (for example, by tilting action to change the direction of motion of the body) it automatically restores itself to the correct position once the disturbance has ceased to exist.

In Fig. 18 is shown a model illustrating a new principle. It takes the form of a two-wheeled motor-car, and has been constructed as an example of a moving body manœuvred by means of a gyrostatic "nose." The model is built up in two parts, a front one, A, and an after and propelling one, B. These are connected together by a vertical hinge, h, an arrangement which to most people will appear, on the face of it, absurd. Rigidly attached to the front part, with its axis horizontal, is a gyrostat, g. The after part carries the propelling mechanism, which is geared up to the back wheel. When the gyrostat is rotating and the car is in motion there is true stability. If the device is started in

the inclined position it erects itself into the upright one. If it is jolted any oscillations which may arise are quickly wiped out.

When the weights m_1 and m_2 shown in the figure are central the action is entirely gyrostatic. The car is, of course, unstable about the line joining the points of contact of the wheels with the ground. Further, the after part is propelling the front part, and thus an instability is introduced through the hinge. These two instabilities without rotation of the flywheel result in complete stability when the car is in motion in the forward direction.

The car is kept upright by the propeller forces. When tilting takes place the gyrostat at once sets about obtaining from the propeller the forces, *and no more*, required to erect the car. The action is shown in detail in A of Fig. 19. The view (1) represents the car when exactly vertical with the two parts in line. The direction of propulsion is indicated by the arrow placed on the after part, and the direction of rotation of the flywheel by the curved arrow. With the direction shown the spin-axis is a straight line, a_1, drawn outwards from the plane of the flywheel towards the reader. View (2) shows the car after a tendency to tilt towards the reader has asserted itself; the tilting couple is represented by a_2. The two parts now get out of line, and the after part exerts on the gyrostat a couple which is represented by a_3. The car now moves up so as to annul the tilting couple. Further, and this is very important, the forward motion of the device tends to bring about a diminution of the want of alignment of the parts. As a result diminution of the tilting couple is accompanied by diminution of the couple applied through the hinge; both couples diminish at about the same rate and finally disappear together. The car is now in the upright position with the two parts in line; its direction of motion has been slightly changed.

Again, suppose the car to be in the upright position with the two parts in line. Let a want of alignment be brought about without there being any tilting couple to account for it (this might be due to a jolt). The gyrostatic action is shown in B of Fig. 19. 1′ shows the car upright with the two parts in line; 2′ shows the device upright, but with the parts out of line; and 3′ shows the car in a tilted position brought about by the want of alignment. The vector a_2' now disappears, leaving a_3', which is accounted for in the manner already explained.

Now, let the gyrostat be rotated in the direction opposed to that in which the wheels of the car rotate, and let the weights

m_1m_2 be put in the positions *aa* of Fig. 18. If tilting takes place the parts turn on the wheels and at the hinge, with the result that the weights *ww* are carried over so as to correct the tilting couple. Further, a stabilising couple is introduced through the hinge. If the weights are placed in the positions *bb* of Fig. 18 the gyrostat should be spun in the direction in which the wheels of the car rotate.

This two-wheeled device is completely stable when being propelled, even though very slowly. When balanced it moves in a perfectly straight path. When the direction of spin is opposite to that in which the wheels of the car rotate the effect of placing a weight to one side—say, to the right, as viewed

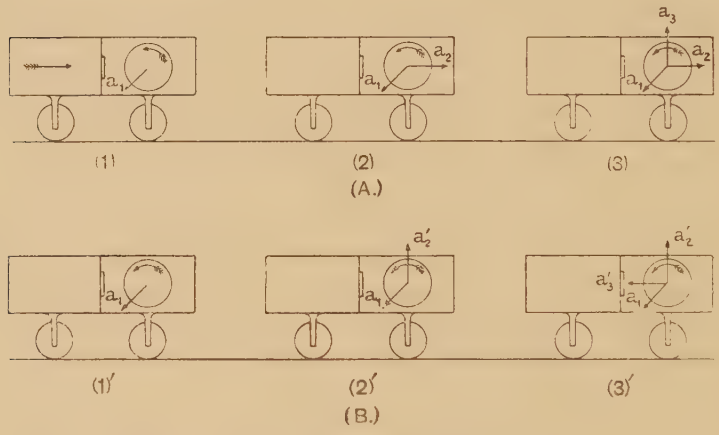

Fig. 19.—Action of Gyrostatic Motor Car.

from behind—is to cause the car to move counter-clockwise in a circular path as viewed from above. Weighting the car to the left causes it to move clockwise in a circular path.

Figs. 20 and 21 show two working models of the device. One of the models, it will be seen, is provided with an electromagnetic steering device. A very small motor, provided with worm gearing, rotates a weight carried at the end of an arm. When the motor is running the weight is carried round in a horizontal circle. When the arm is in line with the car the latter moves in a straight path; when it is to one side the car moves in a circular path in one direction; rotating the weight to the other side of the car and switching off the motor leaves the device moving in a circular path in the opposite direction.

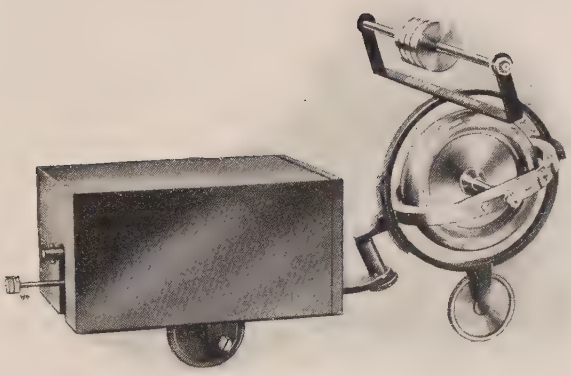

Fig. 20.—Gyrostatic Motor Car.

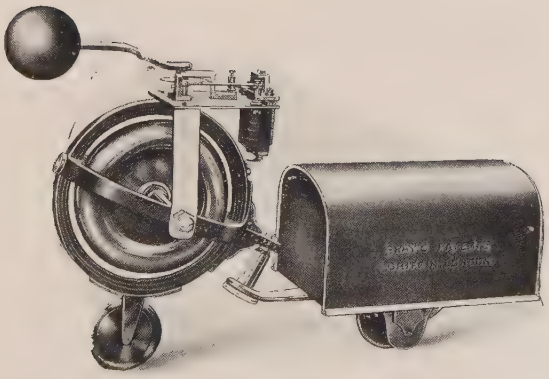

Fig. 21.—Gyrostatic Motor Car with Electromagnetic Steering Device.

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The principles which have just been demonstrated appear of great importance in their applications to airships. As a first example, consider an airship built in two parts, a front one and an after one, connected together by a vertical (or inclined) sleeve. On the front part let there be a gyrostat mounted so as to be laterally unstable without rotation of its flywheel. The after part carries the car, propelling mechanism and power appliances. Provided that the gyrostat is sufficiently powerful, and the construction is properly attended to, this gyrostatic arrangement is one of stability when the airship is in motion. The airship could be manœuvred forcibly in a horizontal plane; the forces required to stabilise the system and to turn the airship against the resistance of the air would be obtained from the propeller system. From the point of view of construction it would simplify matters if the entire airship were made laterally unstable.

Again, let the gyrostat be mounted on gimbal rings at the front of an airship, with its axis fore and aft, and so that the intersection of the gimbal axes passes through the centre of gravity of the gyrostat. Further, let the gyrostat be provided with a system of vanes and springs, so that it is made doubly unstable, without rotation of its flywheel, when the airship is moving in the forward direction. With the flywheel in rapid rotation the gyrostat would be completely stable, and would be of great service in manœuvring the airship in both the vertical and the horizontal directions, the couples bringing about the turning being derived directly from the propelling system, which applies a direct push. To manœuvre the airship couples are directly applied to the gyrostat.

Such gyrostatic airships on a large scale could only be brought to perfection as a result of experiment and trial. Experiments of the nature required are not posssible to a private individual. It may be said, however, that careful calculations have been made relating to the size and power of the motor-gyrostats which would be required, and these could certainly be produced. Such airships are perfectly safe if the gyrostat breaks down. With the propellers reversed and the gyrostat out of action the arrangement is one of stability. In conclusion it may be said that airships on a small scale, capable of being manœuvred by gyrostatic action, could be easily evolved. Such a contrivance could be caused to soar upwards or downwards, or to turn in a horizontal plane in either direction by means of forces derived from a propeller system exercising a direct propulsive force on the system.

ABSTRACT.

The Paper dealt with a number of new contrivances for stabilising, steering and forcibly manœuvring moving bodies, such as torpedoes and airships.

A number of old experiments were first shown. These included the "gyrostat on stilts" and "gyrostat on gimbals" experiments due to Lord Kelvin, the "crossed bifilar" experiment due to Prof. Blackburn, and a stilt top devised by Prof. Harold Wilson. It was shown that the gyrostatic system in each of these experiments, although exhibiting considerable balancing power, was not possessed of real stability. An unstable body rendered truly stable by gyrostatic action must possess the property that if displaced from the mean position it returns to, and comes to rest in, that position. The mean position is that in which the potential energy of the gyrostatic system is a maximum, and if the system is disturbed energy must be supplied to restore it to the mean, or undisturbed, position.

A number of new gyrostatic models were displayed in action. These include two-wheeled and four-wheeled gyrostatic motor cars and bicycles. These all provide examples of gyrostatic systems provided with complete or real stability, and in all the cases shown the stabilising forces are derived from the propelling system.

One of the cars shown runs on two wheels in tandem, and is stabilised by a single gyrostat. This gyrostat is mounted in the car and controls the steering mechanism; it forms, in fact, a gyrostatic chauffeur. The model illustrated a new form of torpedo and airship control.

A second form of motor car, which also runs on two wheels in tandem, consists of two parts, a front one and an after one. The front part carries a gyrostat, the back part the propelling mechanism, and the two parts are connected together by means of a vertical hinge. The front part is propelled by the back part, and the arrangement is one of complete stability. The entire system may be manœuvred by means of the gyrostat. It was pointed out that by properly fitting an airship with a gyrostatic "nose" it should be possible to manœuvre forcibly the airship by means of forces derived from the propellers.

The bicycles, which are provided with gyrostatic riders, are examples of moving bodies steered by gyrostatic action. The action is quite different from that of an ordinary bicycle. They are not "momentum" instruments.

The devices shown are at once applicable to long-distance torpedoes, both submarine and aerial. The gyrostatic system may be operated by the wireless transmission of electrical action.

At the conclusion of the Paper the author showed a new series of animated gyrostats.

DISCUSSION.

Dr. W. Watson thought the mechanisms shown were of great theoretical importance. He gathered, however, that the author himself thought they were more of theoretical than practical interest. He concluded some time ago that a two-wheeled car would not be of much use, as, although gyrostatic control worked satisfactorily either on a straight path or on a curved path of constant curvature, any attempt to alter the curvature had to be made with great caution. Hence a train built

on this system would have to slow up on approaching either the beginning or the end of a bend. With a motor car, where one had to steer immediate courses on account of other traffic, the arrangement would be impracticable. At one time, when some cars had engines laid longitudinally and others transversely, makers of the latter type claimed that gyrostatic action came into play and tended to prevent skidding. However, unless the gyrostat was free to move relatively to the car, one might as well have a lump of iron in its stead. He had investigated the amount of relative motion which might take place due to give in the springs or mountings, and it was quite insufficient to allow of appreciable gyrostatic action.

Mr. DUDDELL complimented the author on the collection of beautiful models which he had brought before the Society and the admirable way in which he had explained the principles underlying their action. He asked what speed was attained by the flywheels of the gyrostats.

Mr. R. S. WHIPPLE also expressed his admiration of the models.

Mr. F. J. WHIPPLE asked if the author had worked out the theory of the ordinary bicycle, and if it was his considered opinion that the rider had to perform the actions which he had described in steering. It was his opinion that when travelling rapidly this was not so, and that there was a stabilising effect due to the gyrostatic action of the front wheel. This was particularly noticeable in the way in which the wheel seemed to be pulled back into position if, when riding without the hands, the cyclist encountered a small stone.

Dr. RUSSELL asked concerning the use of the word "gyrostat." He remembered on one occasion when Lord Kelvin was showing some of these experiments to von Helmholtz an accident occurred which resulted in one of the gyroscope wheels passing through Helmholtz's silk hat. After that it was customary to enclose the gyroscope in a brass case, and it was then usually called a gyrostat. He did not quite see why.

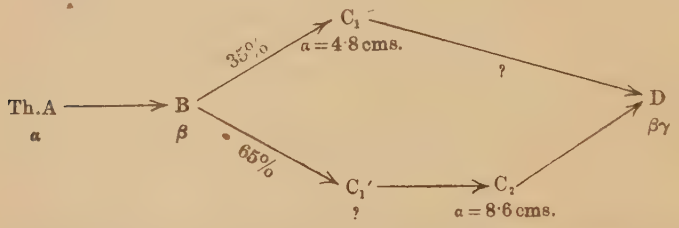
The AUTHOR, in reply, said that the larger gyrostats could be taken up to a speed of 20,000 revs. per min. in half a minute, and would run for 75 minutes. In steering a bicycle the rider turns the front wheel to the side to which the machine leans, and the forward momentum brings it up to the vertical position. The gyrostatic action helps, but only to a very slight extent. The name "gyrostat" was the one invariably used at Glasgow since Lord Kelvin's time.

XXVI. *Volatility of Thorium Active Deposit.* *By* T. Barratt, *A.R.C.S., B.Sc., and* A. B. Wood, *M.Sc.*

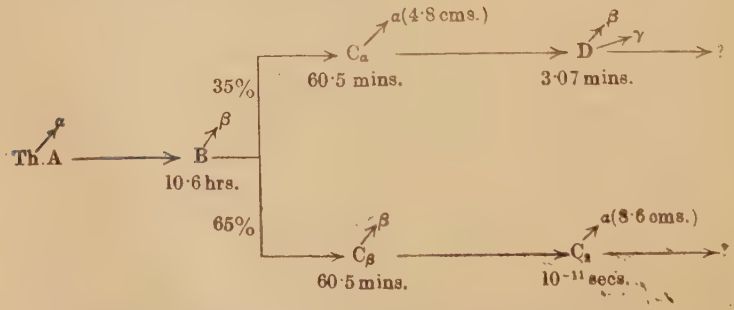
Received April 21, 1914.

I.—Introduction.

In a Paper published in 1911 by E. Marsden and one of us,* it was first proved that the two α-ray products contained in ThC are not successive, but belong to different branches of the thorium series. The following scheme of disintegration was suggested :—



The hypothetical product C_1' was included in order to account for the fact that C_1 and C_2 appeared to be always present in the same relative proportions, viz., 35 : 65. This could not possibly be the case if C_2 came directly from B, unless C_2 had the same period as C_1. This, however, is unlikely, as the range of C_2 is nearly double that of C_1, and in the case of most other α-ray products of any particular radio-active family, Geiger and Nuttall's rule that log (transformation constant), log (range) is a constant applies pretty closely. Marsden and Darwin,† after further experiments with thorium active deposit, proposed the scheme :—



* Marsden and Barratt, "Proc." Phys. Soc., 24, p. 50, 1911.
† Marsden and Darwin, "Proc." Roy. Soc., A, LXXXVII., p. 17, 1912.

In this arrangement C_α and C_β are supposed to represent, not two different products, but the same product breaking down in two different ways, one with the expulsion of an α-particle, giving rise to D, the other emitting a β-particle, and forming C_2, which in its turn is transformed to an unknown substance on expelling an α-particle.

Miss Meitner,* however, obtained results which seemed to indicate that the hypothetical substance C_1' (or C_β) could be obtained free from C_1 (or C_α). On adding a few drops of stannous chloride to an HCl solution of the active deposit, and dipping successive nickel plates in the solution, the C_α appeared to be removed, leaving C_β and D in solution. The latter was then boiled to dryness, divided into two portions, and, the D being first allowed to decay, activities measured by α and γ-rays respectively. The γ-curve rose from zero, showing that C_α, and therefore also D (which produces the γ-rays), had been removed by the nickel plates. The α-radiation, however, arising presumably from C_2, did not commence from zero, showing that C_β (with C_2) had not been completely removed.

Marsden and Wilson† repeated Miss Meitner's experiments, and acknowledged obtaining sometimes the same results, but concluded on the whole that they were anomalous, and due mainly to the fact that, in the presence of HCl, C volatilises at a comparatively low temperature. In their experiments the active deposit was heated to temperatures not accurately measured, but estimated at 250°C. to 300°C.

In view of these conflicting results, it was thought desirable to attempt to separate the members of the active deposit by heating the latter to various accurately measured temperatures, and following the consequent changes of activity, making measurements of both α and β-radiations.

II.—Previous Work on Volatilisation.

The first attempt to separate the members of the thorium active deposit by volatilisation appears to have been made by Miss Gates‡ in 1903. She obtained apparently a partial separation of B and C (then known as A and B respectively). Two years later Miss Slater§ heated for one or two minutes a

* Lise Meitner, "Phys. Zeit.," XIII., p. 623, 1912.
† Marsden and Wilson, "Phil. Mag.," pp. 354-361, Aug., 1913.
‡ Miss Gates, "Phys. Rev.," 16, p. 300, 1903.
§ Miss Slater, "Phil. Mag.," 9, 628, 1905.

platinum wire coated with the active deposit to temperatures up to about 1,300°C. B commenced to volatilise at 630°C., but no part of C was removed till a temperature of 730 deg. was reached. At 1,280°C. the whole of B was removed, and 99 per cent. of C, the small remaining activity (presumably measured by α-radiation) diminishing with a period of about one hour. The temperatures in these experiments were measured by a thermo-junction. No β-ray measurements were made, so that the results do not throw much light on the question of the separability of the various products now known to be present in the thorium series.

III.—Apparatus and Measurements.

(*a*) *Active Deposit Undissolved in Acid.*

An electric furnace, consisting of a porcelain tube wound with strip platinum, was employed for heating the active deposit to any required temperature. The internal dimensions were, approximately, length 60 cm., diameter 4·5 cm., this large size ensuring that the volatilised portion of the deposit could easily escape, and would have little chance of settling once more on the plate on which it was originally deposited. The temperature was accurately measured by a platinum thermometer, contained in a tube of biscuit porcelain, in connection with a Callendar-Griffiths' "bridge," whose coils were carefully calibrated in the usual way. The "ice" and "steam" points were determined, and verified from time to time. The δ-coefficient of the platinum was taken as 1·50. In order to ensure that the temperature of the active deposit was that indicated by the thermometer, the active plate was placed in a small flat platinum "basket," which was hung on the thermometer near its extreme end. The active deposit was obtained from a preparation of mesothorium, very kindly lent by Mr. F. H. Glew. By a simple arrangement the active deposit could be obtained on one side only of a platinum foil, and by a suitable adjustment of the conditions of exposure considerable variations in its activity could be obtained. In earlier experiments the results were somewhat variable, owing to the fact that the platinum used was not perfectly clean. The slightest trace of grease, &c., is enough to spoil an experiment, as the active matter is carried away when the grease evaporates. (This may possibly account for the comparatively low temperatures of volatilisation obtained by Miss

Slater.*) Consequently the foils were strongly heated for some time in a blow-pipe flame before being exposed to the emanation. To check results, the active plates were frequently placed in the furnace in pairs. It was also found advisable not to use an electric field when exposing the plates to the emanation, as in such cases a small percentage of ThX was usually found with the active deposit. This precaution also precluded the possibility of any appreciable quantity of active deposit from radium, which is always present in mesothorium, being attracted to the plates. In addition, an interval of at least five hours was allowed to elapse—after removal from the emanation—before any measurements of activity were made. The period of decay was then governed by that of ThB (10·6 hours). Small pieces of quartz were used in some experiments instead of platinum, but no difference was observed in the temperatures of volatilisation. This result is different from that obtained by Makower† in the case of radium active deposit. In every experiment measurements of the activity of the foil were made for some time before it was placed in the furnace, an α or β-ray electroscope being used for the purpose.

(i.) *Measurements by α-rays.*

The deposit was heated to the required temperature, in most cases for 15 minutes, as the volatilisation appeared to be to some extent a time effect, and immediately after removal from the furnace the activity was again measured, and continued for several hours, until the decay was exponential, with the period of ThB.

Fig. 1 illustrates graphically the results of one of these experiments. The first part (A) gives the curve of activity before the plate was heated, the period of decay then being that of B. The active plate was in the furnace for a time indicated by *a*—*b*. After removal from the furnace, the curve of activity was much steeper, as is shown in the latter portion (B) of the curve. In order to calculate the percentage of C-activity removed by heating, the curve B was produced back to the point Q on the line *Pc*, which corresponds to the time when the plate has been in the furnace four minutes. Previous observations had indicated that in this time, at a given constant temperature, practically the maximum amount of active matter has been volatilised. The percentage of C removed is then

* Miss Slater, *loc. cit.*

† Makower, "Le Radium," 6, p. 50, 1909.

given by 100 PQ/Pc. To calculate the percentage of B removed, readings were taken six or seven hours after the foil was heated. The values then observed were corrected (to the time of heating) for decay of B (period 10·6 hours), and the percentage reckoned as in the case of C.

Fig. 2 embodies the results of a great number of experiments of a similar kind at various temperatures, the measurements

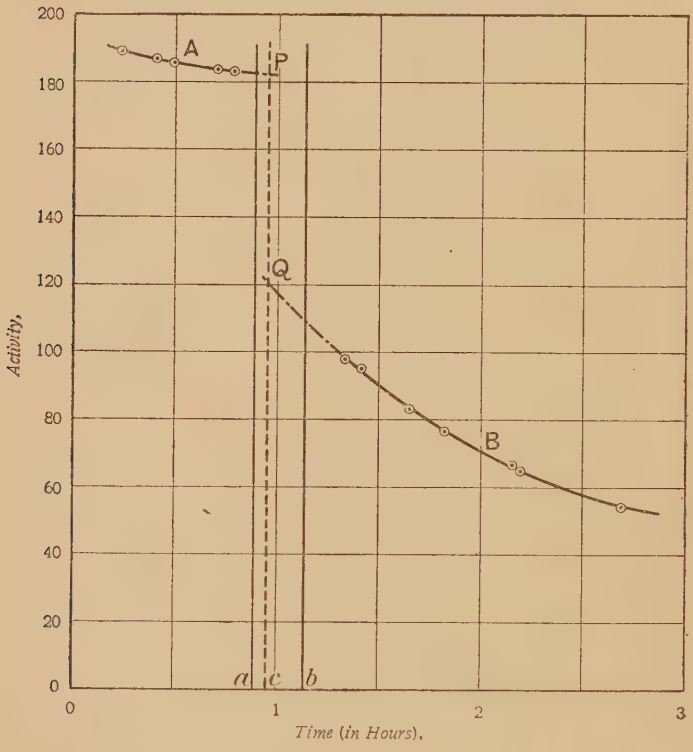


Fig. 1.

all being of the α-radiation. It can easily be deduced from the curves that :—

1. Both B and C begin to volatilise at about 750°C. :—B perhaps at a temperature slightly lower than C.

2. Between 750°C. and 1,200°C. a greater percentage of B than of C is volatilised at any given temperature.

3. Volatilisation of both B and C is practically complete at 1,200°C.

4. There is an inflexion of the C curve between 750°C. and 900°C; in fact the curve is similar to two of the " B " curves placed end to end. This points to the possibility that between 750°C. and 900°C. only one α-ray product is being volatilised; after 900°C. the curve becomes very much steeper, as if the second product is also being driven off.

5. The inflexion occurs at a point where about 35 per cent. of the α-ray activity has disappeared.

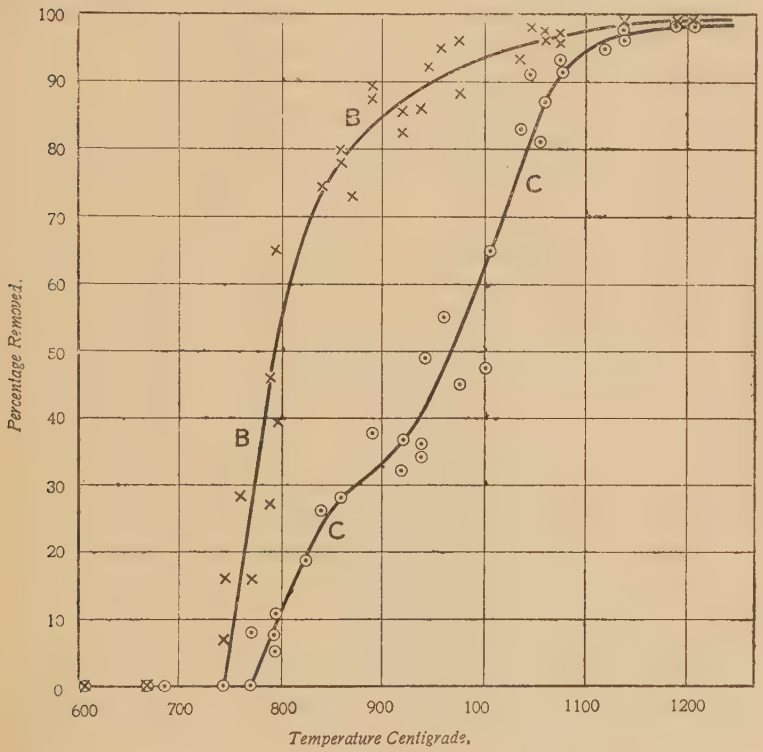


FIG. 2.

(ii.) *Measurements by β-rays.*

In view of the fact, before mentioned, that the ratio of C_α to C_2 is as 35 : 65, t was recognised that the shape of the C curve was significant. In order to throw more light on the question similar experiments were conducted, employing this time the

β-radiation. The result of one of these experiments is exhibited in Fig. 3. It is noticeable that :—

1. There is at first a steep rise of the curve, undoubtedly due to the growth of D, which is known to be at any rate more volatile than B or C.*

2. The curve then descends with a period a little greater than that of C.

3. The activity after several hours decays with the period of B.

As the result of many similar experiments at various temperatures it was found that, *as measured by β-rays*, C does not begin to volatilise till a temperature of about 900°C. is reached. It

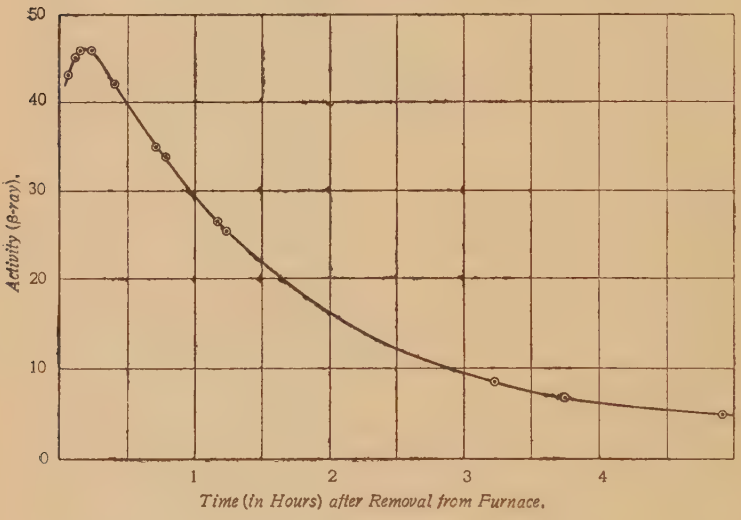


Fig. 3.

is significant that this temperature, again, is that at which the second part of the C curve, as obtained by α-ray measurements, commences. An explanation of these observations will be attempted in section IV.

(*b*) *Experiments with the Active Deposit Dissolved in Acid.*

As some of the researches quoted† were carried out with the active deposit dissolved in acid, some experiments were done with solutions of the active substance in pure conc. HCl, or

* v. Lerch and v. Wartburg, "Wien. Ber.," 118, p. 1575, 1909.

† Marsden and Wilson, Miss Meitner, *loc. cit.*

HNO_3. As a result it was found that in the case of the HNO_3 solution volatilisation occurred at practically the same temperatures as before. With HCl, however, the activity disappeared at a much lower temperature, commencing, in fact, at about 300°C. In the case of solution in HNO_3, as with the undissolved product, the deposit is probably in the form of an oxide, while the solution in HCl is no doubt a chloride. In this connection some results of Dr. Schrader's* are interesting. He found that with the active deposit of actinium, volatilisation occurred at a much lower temperature if a platinum wire coated with it were first exposed to an atmosphere of chlorine. Russell† also discovered that in hydrogen RaC was volatile at 360°C., while in oxygen the temperature had to be much higher. In Marsden and Wilson's Paper, again, it is shown that ThC volatilises at about 300°C. when dissolved in HCl.

(*c*) *Volatility of Thorium D.*

As the result of experiments similar to that exhibited graphically in Fig. 3 (β curve), it was concluded that D begins to volatilise at about 500°C. The active deposit, on a platinum foil, and undissolved in acid, was heated for 15 minutes in the furnace at various temperatures. Up to 500°C. there was no break at all in the curve of activity as obtained by β-ray measurements. At this temperature, however, some of the D was removed (as was indicated by the initial rise of the curve after removal from the furnace). The same phenomenon was observed at higher temperatures, up to about 900°C. Beyond this temperature, however, provided the active plate was in the furnace long enough (*e.g.*, 10 or 15 minutes) there was no perceptible rise of β activity due to growth of D. Assuming that C_α is removed almost entirely at 900°C., while C_β is not removed to any extent, this verifies the experiments of Marsden and Darwin,‡ and of one of us,§ that D arises entirely from the 35 per cent. branch, viz., C_α.

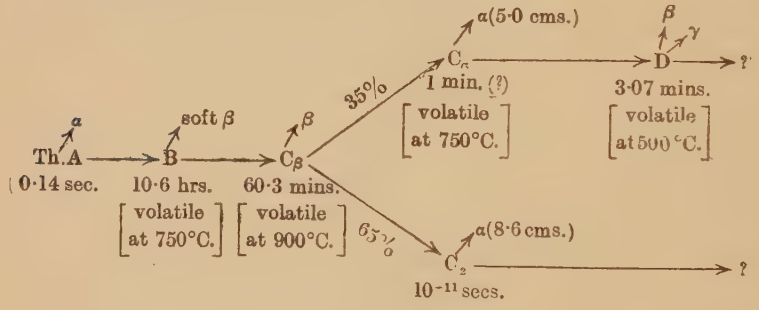
IV.—Discussion of the Results.

The fact that volatilisation of both B and C commences at 750°C., and is not complete until about 1,200°C., precludes the possibility of an absolute separation of these two products

* Schrader, "Phil. Mag.," 24, p. 125, 1912.
† Russell, "Phil. Mag.," 24, p. 134, 1912.
‡ Marsden and Darwin, *loc. cit.*
§ A. B. Wood, "Phil. Mag.," pp. 586-597, Oct., 1913.

from each other by this method. This wide range of temperature seems rather remarkable. It may be that the oxides are gradually reduced to the pure metal, which is not volatilisable except at a very high temperature. The case may be, in fact, similar to that of mercury, which, when heated in air, oxidises at a comparatively low temperature, and is again reduced to pure mercury at a considerably higher one. A temperature greater still is then required to vaporise the metal.

In order to explain the results already quoted in the present Paper it is evident that some modifications must be made in schemes of disintegration that have been proposed up to now. The following arrangement appears to us to fit the facts better than any other we can devise :—



It will be observed that C_β is assumed to be a separate product, which breaks up in two ways, 35 per cent. of the disintegration producing C_α and 65 per cent. giving C_2. As in the case of B and C (the latter taken as a whole), there is only a partial separation of the two products, but evidently C_α is more readily volatile than C_β.

The scheme accounts for :—

1. The branching of C_α and C_2.

2. The differences in volatility obtained in α and β-ray measurements respectively. Neglecting the small percentage of β-radiation from D, which can easily be identified and allowed for, all the β-activity measured in the present experiments comes from C_β, as the base of the electroscope employed was too thick to be penetrated by the soft β-rays from B. The experiments show then that C_β is not removed to any perceptible extent until a temperature of 900°C. is reached, while C_α begins to volatilise at 750°C.

3. The periods of B and C (10·6 hours and 60·3 minutes

respectively). On the above scheme the period of C is governed by that of C_β. This assumes, of course, that the period of C_α is comparatively small (perhaps only a minute or so). This will be discussed later.

4. The inflexion in the curve of volatilisation of C. This has been already mentioned. (*See* Section III.)

5. The constant ratio of the activities of C_α and C_2. Several experiments were tried in the present research in order to test the constancy of this ratio. Measurements were taken of the activity of the plates before and after heating :—

(*a*) With active plate uncovered ;

(*b*) With plate covered with just sufficient aluminium foil to stop the α-rays of range 4·8 cm.

The ratio of activities (before and after heating) was about the same in both cases (*a*) and (*b*). This result, however, is easily explained if we make the assumption suggested in (3) above, *i.e.*, that the period of C_α is no more than a minute or so. It must be remembered that by far the greater proportion of the α-activity is produced by C_2 in virtue of—

(*a*) Its longer range (8·6 cm. to 4·8 cm.).

(*b*) The fact that it gives twice as many α-particles.

(*c*) It produces its effect in both cases, whether the foil is covered or uncovered.

It would therefore be difficult to prove the existence or otherwise of a small amount of C_α. In any case, assuming that C_α has a short period, the above scheme shows that C_α and C_2 would each be in equilibrium (in the ratio of 35 to 65) with the common parent product C_β in a very short time—almost, in fact, before any reliable measurements of the relative α-activities could be made.

As a great deal in the explanation given depends on the life of C_α being considerably shorter than that usually attributed to C as a whole (viz., 60·3 minutes), it is of importance to note that, in order to fit the relation given by Geiger and Nuttall,* the period of C_α (with its range 4·8 cm.) should be only two or three minutes. The range usually attributed to C_α (4·8 cm.), was determined by Marsden and Barratt† by a scintillation method, which is well known to give lower results than the usual ionisation determinations.‡

* Geiger and Nuttall, " Phil. Mag.," 22, p. 613, 1911 ; 23, p. 439, 1912.

† Marsden and Barratt, *loc. cit.* ; Barratt, " Proc." Phys. Soc., 24, p. 112 1912.

‡ Rutherford, " Radioactive Substances and their Radiations," p 161, 1913.

If the range were taken as 5·0 cm. (as given by some experimenters),* then Geiger and Nuttall's relation would assign a half-period of less than a minute to C_a.

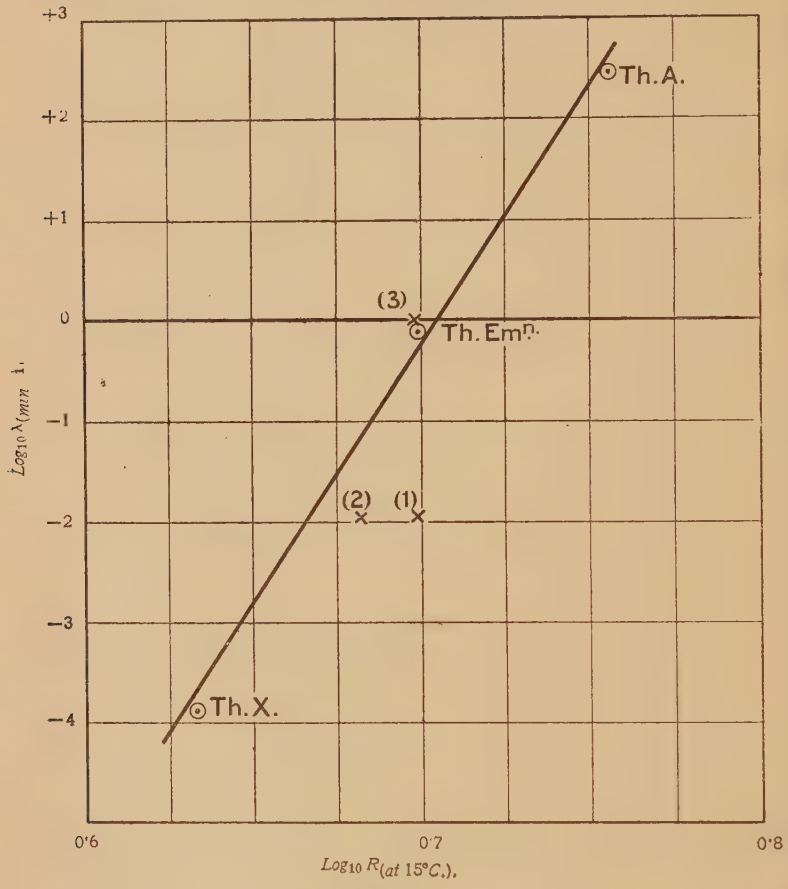


FIG. 4.

This is clearly shown in Fig. 4, the crosses indicating the positions that would be occupied by C_a if its half-period and range were respectively—

1. 60·3 mins. ; 5·0 cm.
2. 60·3 mins. ; 4·8 cm.
3. 40 secs. ; 5·0 cm.

* Hahn, "Phys. Zeit.," 7, p. 456, 1906.

Experiments are in progress with the object of further elucidating the points raised above. The radium and actinium series are also being tested in a similar way; and by heating these products in atmospheres of different gases it is hoped that further knowledge may be gained of the separability, &c., of the members of these series.

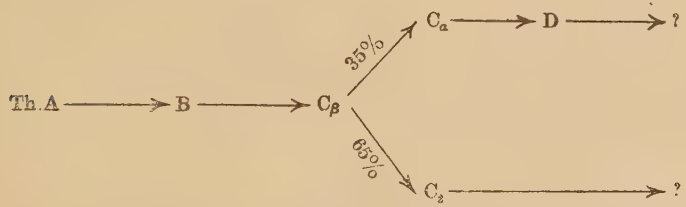
Summary.

1. By heating thorium active deposit to various temperatures up to 1,250°C., it is found that B and C each begin to volatilise at 750°C. (when measured by α-radiation), but volatilisation is not complete until a temperature of about 1,200°C. is reached.

2. The curve of activity of C is similar to two "B-curves" in series, the inflexion occurring at about 900°C., where about 35 per cent. of the α-activity has been removed.

3. When measured by its β-activity, C volatilises at 900°C. and D at 500°C.

4. In explanation of these results, it is assumed that C_β is a separate product, which breaks up in two ways, each with the expulsion of an α-particle, producing C_α and C_2 as shown in the following scheme :—



5. Temperature of volatilisation "in HNO_3" same as ordinary active deposit. Temperature of volatilisation in HCl, 300°C. for Th.B and C.

The experiments have been carried out at the Woolwich Polytechnic, and our best thanks are accorded to the Principal and Governors of that institution; also to Mr. F. H. Glew, who very kindly lent us, for a considerable period, the mesothorium used in the research.

ABSTRACT.

On heating thorium active deposit to various accurately measured temperatures up to about 1,250°C., it is found that B and C each commence to volatilise at 750°C., but the volatilisation is not complete until 1,200°C. is reached, the measurements being made by an α-ray electroscope. The C curve is peculiar, being similar to two of the

B curves placed end to end, the inflexion occurring between 750°C. and 900°C., where about 35 per cent. of the α-activity has been removed. When measured by β-radiation, C is not volatile until a temperature of 900°C. is reached. D commences to volatilise at 500°C.

To explain these results, it is assumed that the part of C which produces the β-rays—viz., C_β—is a separate product, which is not so readily volatile as C_α. A suitable scheme of disintegration is suggested.

DISCUSSION.

Dr. R. S. Willows congratulated the authors on their able treatment of the subject. The suggested scheme met most of the requirements of the case and was not in contradiction with any of the other known properties of the thorium series. He hoped the authors would extend their work to other radio-active series.

Dr. S. Russ drew attention to the apparent lack of parallelism between the characteristics of the thorium and radium series. The volatility of thorium B appeared to be not very different from that of thorium C, whereas in the radium series, radium B is the most, and radium C the least, volatile of the series. Again, the authors concluded that there was no appreciable difference in the volatility of the thorium active deposit from a quartz or a platinum surface, whereas there was an appreciable difference in the case of the radium active deposit as Dr. Makower had shown.

Mr. D. Owen observed that one of the products was stated to have a period of 10^{-11} second. He thought it hardly possible to catch a product which only lived for that time.

Mr. Barratt said that only two surfaces were used, quartz and platinum, on account of the high temperatures to which they had to be subjected. In these cases there had certainly been no difference. When different acids were used, considerable differences were observed in the temperature of volatilisation. The period, 10^{-11} second, assigned to Th C_2 was simply in order to fit the Geiger-Nuttall relation. The curve connecting log (χ) and log (Range) is a straight line, and the period 10^{-11} second was necessary to make the product fit the curve.

FIG. 1.

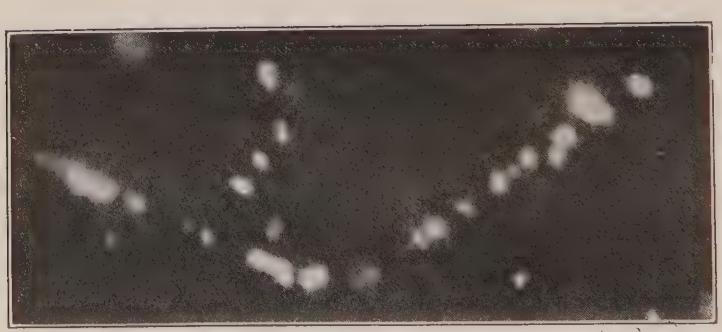


FIG. 2.

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XXVII. *The Passage of α-Particles through Photographic Films.*

By H. P. WALMSLEY, *M.Sc., and* W. MAKOWER, *M.A., D.Sc.*

RECEIVED APRIL 17, 1914

THE photographic action of α-particles was first studied in detail by Kinoshita,* who showed that, whenever an α-particle, strikes a grain of silver halide in a photographic plate, that grain is subsequently capable of development. It was also shown that this was the case throughout the range of the α-particle in the gelatin. It therefore seemed probable that the path of an α-particle, projected tangentially to a photographic film, would, after developing the plate, be seen under a microscope as a trail of grains. This effect has been observed by Reinganum† and the method has been employed by Michl‡ and Mayer§ to detect α-particles ; but microphotographs demonstrating the phenomenon do not seem yet to have been obtained. In some experiments upon which we were engaged, it became necessary to examine microscopically photographs taken with very weak sources of α-radiation and it occurred to us that it might be of interest to obtain microphotographs of the trails which could be clearly seen in the films.

As a nearly pure source of α-radiation of fairly long range, a flat metal plate was exposed for a few seconds to a fraction of a millicurie of radium emanation and then immediately placed in contact with an Ilford process plate. The α-rays from the radium A on the metal surface produced a photographic impression on the plate which was just visible to the naked eye after development. Under the microscope this faint impression was seen to consist of a multitude of definite tracks passing through the film at all angles. A microphotograph of the film taken with a $\frac{1}{3}$ in. objective is reproduced in Fig. 1. It was found best to use this relatively low-power objective for, as the grains of silver affected by the α-particles were at different depths in the film, it was impossible to focus all the grains simultaneously when using an objective of higher power. The photograph confirms in a striking manner the conclusions

* Kinoshita. " Proc." Roy. Soc., A83, 432, 1910.
† Reinganum, " Phys. Zeits.," 12, 1076, 1911.
‡ Michl, " Wien. Ber.," 2A, 1431, 1912.
§ Mayer, " Ann. d. Phys.," 41, 5, 931, 1913.

drawn by Kinoshita which have been mentioned above. Careful examination of Fig. 1 shows that many of the α-particles, after traversing the film in straight lines for some distance, suffer sudden deflections, thus illustrating the well-known effect of scattering; but the great number of the tracks on this photograph and the smallness of the magnification make it difficult to follow out these details easily. Moreover, the α-particles penetrate the film in all directions, so that only a relatively small number leave tracks sufficiently nearly parallel to the plane of the film to be examined throughout their lengths. On this account the photographic impression produced just outside the edge of the active plate was examined; for in this region the density of the tracks is much smaller and a greater proportion of the α-particles producing them were travelling parallel to the plane of the film. A microphotograph similar to that shown in Fig. 1 was taken of this region of the photographic plate and a few isolated tracks were again magnified by photography with an ordinary camera. Three such tracks are shown in Figs. 2, 3 and 4, which show the phenomena in greater detail. In the cases of the tracks shown in Figs. 2 and 3 the magnification was measured by determining the lengths of the paths on the original photographs with a graduated scale in the eyepiece of a microscope. The magnification in Fig. 2 was found to be 1,650 diameters; that in Fig. 3 was 1,800. It will be seen that the magnification in Fig. 4 is slightly less. The large spots seen on Figs. 3 and 4 near the tracks are due to minute particles of dust on the original photograph.

Some difficulty was experienced in making sure that the tracks showing deflections were due to single α-particles and not to two particles passing through the film in different directions. There is no means of deciding this point from a study of the microphotographs, but it is an easy matter to trace out the complete path of an α-particle throughout its length by visual observations through a microscope. For as a rule the grains in any track are not precisely at the same depth in the film and it is possible by careful focussing to ascertain whether an α-particle has been steadily penetrating into the film or emerging from it. In cases in which two tracks crossed each other, at the point of apparent intersection the grains due to the two α-particles were in general seen to be at quite different depths in the film.

This method of studying the projection of α-particles through matter shows many of the characteristics demon-

Fig. 3.

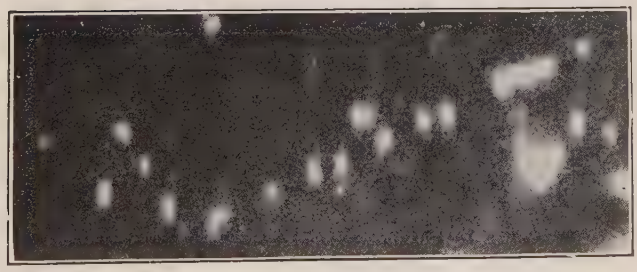

Fig. 4.

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strated by C. T. R. Wilson* in his beautiful photographs of the tracks of α-particles through air. The method just described possesses, however, the advantage of great simplicity, and may prove of use in studying the scattering of α-particles produced by heavy substances, such as silver.

ABSTRACT.

It has been shown by Kinoshita that when an α-particle strikes a grain of silver halide, that grain is subsequently capable of photographic development. It therefore seemed probable that the path of an α-particle projected tangentially to a photographic film should, after development, be visible under a microscope. This was shown to be the case, and micro-photographs showing the tracks of α-particles through a photographic plate have been obtained. The effect of "scattering" of α-particles can also be seen in the photographs and this method may prove of use in studying the scattering of α-particles by heavy atoms such as silver. This method of studying the path of an α-particle possesses the advantage of great simplicity.

* C. T. R. Wilson, "Proc." Roy. Soc., A87, 277, 1912

XXVIII. *On a Null Method of Testing Vibration Galvanometers.* *By* S. BUTTERWORTH, *M.Sc., Lecturer in Physics, School of Technology, Manchester.*

RECEIVED APRIL 11, 1914.

1. In a recent Paper A. Campbell* has given the constants of a number of vibration galvanometers. In obtaining these constants the measurements made were the alternating-current and voltage sensitivities, the direct-current sensitivity and the resonating frequency. Of these, the alternating-current and voltage sensitivities are the most difficult to measure, since, because of the sharpness of resonance, there is a considerable fluctuation in the amplitude of vibration for very small variations in the frequency of the source, while the presence of any harmonics in the source will introduce a definite error in the results.

In the course of an investigation on electrically-maintained vibrations, the author has succeeded in developing a null method whereby the vibration constants of a galvanometer may be determined and in which the harmonics in the source have no effect. Measurements by this method, together with a knowledge of the direct-current sensitivity, are sufficient to determine the galvanometer constants.

The principle of the method depends on an extension of the theory of the vibration galvanometer. It is shown that, as far as the electrical behaviour of the instrument is concerned, we may regard it as built up of a parallel combination of a capacity, an inductance and a resistance, in series with a resistance.

At resonance, the capacity and inductance neutralise each other and the galvanometer then behaves as a pure resistance. The value of this resistance is the effective resistance as measured by Campbell.

A direct measurement of the effective resistance at resonance by a null method, is, however, practically impossible, since fluctuations in the frequency of the source have their maximum effect on the impedance of the galvanometer in this state.

It will be seen, however, that by a suitable arrangement of inductances and capacities a complete balance can be obtained

* "Proc." Phys. Soc., Feb., 1914.

for any form of current, and that the method will be applicable, with certain limitations, to all the galvanometers tested by Campbell.

The units employed will be those of the C.G.S. electromagnetic system, unless where otherwise stated. For the physical interpretation of the symbols reference should be made to Campbell's Paper. The present notation as compared with his is as follows :—

α	β	γ	A	r_v	r	p	p_0	k
mk^2	b	c	$10g$	R′−R	R	ω	ω_1	$\frac{2}{h}\times 10^{-4}$

The symbols in the second line are those used by Campbell.

2. Using the notation employed in a previous Paper,* the equation of motion of the coil is

$$\alpha\frac{d^2y}{dt^2}+\beta\frac{dy}{dt}+\gamma y=\mathrm{A}i, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where y and i are the instantaneous values of the deflection and current respectively. If the galvanometer has ohmic resistance r and inductance l, the E.M.F. across the galvanometer terminals is

$$e=ri+l\frac{di}{dt}+\mathrm{A}\frac{dy}{dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the last term being the back E.M.F. due to the motion of the coil.

When the current is alternating with a frequency $p/2\pi$, equations (1) and (2) transform into the vector equations

$$\left.\begin{aligned}(\gamma-\alpha p^2+jp\beta)\mathrm{Y}&=\mathrm{AI}\\ \mathrm{E}&=(r+jpl)\mathrm{I}+jp\mathrm{AY}\end{aligned}\right\}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where Y, I, E are the vectors representing the vibration current and E.M.F. respectively. Eliminating Y

$$\mathrm{E}=\left(r+jpl+\frac{jp\mathrm{A}^2}{\gamma-\alpha p^2+jp\beta}\right)\mathrm{I}. \quad . \quad . \quad . \quad . \quad (4)$$

The quantity in ( ) is the vector impedance of the vibration

* Butterworth, "Proc." Phys. Soc., Vol. 24, p. 75, 1912.

galvanometer, the last term being that contributed by the vibration. If we put

$$\beta/A^2=S_v=\frac{1}{r_v},\quad \alpha/A^2=C_v,\quad A^2/\gamma=L_v,\ .\ .\ .\ (5)$$

then the vector impedance becomes

$$G=r+jpl+\left\{S_v+j\left(pC_v-\frac{1}{pL_v}\right)\right\}^{-1}.\ .\ .\ .\ (6)$$

The inductance (l) of the galvanometer can be neglected in practice, so that with this assumption we see that the galvanometer is equivalent to a parallel combination of a conductance (S_v), a capacity (C_v) and an inductance (L_v) in series with the ohmic resistance (r).

If $$p^2=\frac{1}{L_vC_v}\equiv\frac{\gamma}{\alpha}=p_0^2\ \text{(say)},\ .\ .\ .\ .\ .\ .\ (7)$$

then $$G=r+\frac{1}{S^v}=r+r_v.\ .\ .\ .\ .\ .\ .\ (6A)$$

This holds when the galvanometer is in resonance with the source, so that (6A) gives the effective resistance at resonance.

If $p\neq p_0$, then for good galvanometers the damping (β) may be neglected, so that, putting $S_v=0$, we have

$$p>p_0 \qquad G=r-j/pC_v\left(1-\frac{1}{n^2}\right),\ .\ .\ .\ .\ (6B)$$

$$p<p_0 \qquad G=r+jpL_v/(1-n^2),\ .\ .\ .\ .\ .\ (6C)$$

where $n=p/p_0$.

Hence, for frequencies of the source above resonance the galvanometer may be treated as a condenser with a series resistance, and for frequencies of the source below resonance as an inductive resistance.

The quantities r_v, C_v, L_v will be referred to as the *vibration resistance*, *vibration capacity* and *vibration inductance* respectively, and they will be called generally the *vibration constants* to distinguish them from the intrinsic constants α, β, γ, A.

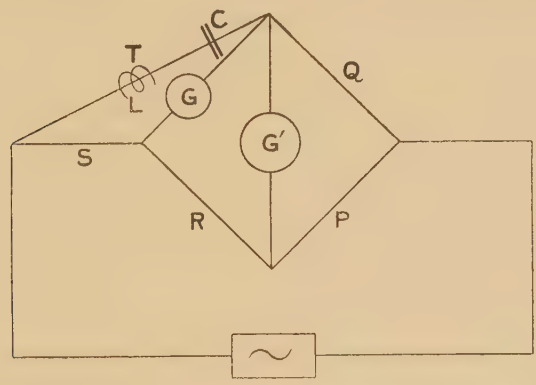
3. *Determination of the Vibration Constants.*

In order to obtain approximate values of L_v and C_v all that is necessary is to measure the apparent inductance or capacity of the galvanometer at two known source frequencies. Then, by means of equations (7), (6B), (6C), the values of L_v, C_v and p_0 can be found. The quantities thus measured depend on the

frequency, so that the source should be free from harmonics and the fluctuations of frequency should be small. If the source frequencies chosen are well removed from the resonating frequency, however, the effect of fluctuations may be neglected.

In order to obtain the vibration resistance, it will be necessary, however, to work with the galvanometer almost in resonance with the source. The effect of the fluctuations in frequency is now very large, so that although equation (6A) holds at resonance no balance will be possible on a direct resistance bridge unless the source is almost perfectly steady. By using the bridge below, however, a balance can be got even with a very impure form of alternating current.

In the figure G is the galvanometer under test, G′ is the detector, P, Q, R, S are non-inductive resistances, and the arm



T is made up of a non-inductive resistance, ρ, an inductive resistance (U, L), and a condenser (C) all in series.

If T, G are the vector impedances of the corresponding arms, the condition of balance is

$$T(GP-QR)=Q\{G(R+S)+RS\}. \quad . \quad . \quad . \quad (8)$$

In this equation T is given by

$$T=\rho+U+j(pL-1/pC)$$

and

$$G=r+\left\{S_v+j\left(pC_v-\frac{1}{pL_v}\right)\right\}^{-1}.$$

Now, let the two following balances be made with *direct* currents :—

(*a*) With the arm T open, adjust P or Q for balance. Then, since $G=r$,

$$rP=RQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

(*b*) With the condenser (C) short-circuited, $\rho=0$, and the arm G open, adjust S for balance. Then

$$UP=(R+S)Q. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

When these conditions are satisfied equation (8) reduces to

$$(T-U)(G-r)=r(U+S), \quad . \quad . \quad . \quad . \quad (11)$$

so that, putting in the values of T and G, the further conditions of balance *for all frequencies* are

$$\frac{\rho}{S_v}=\frac{L}{C_v}=\frac{L_v}{C}=r(U+S). \quad . \quad . \quad . \quad . \quad . \quad (12)$$

In attaining these conditions in practice the values of L_v and C_v are already known approximately, so that the values of L and C are thus roughly fixed. Also L_v depends on γ (*see* equation 5), which in turn is altered by the tuning of the galvanometer. Hence, by using a variable self-induction for L, all the adjustments may be made without disturbing the direct-current balances (9) and (10).

Hence, complete balance for alternating currents can be got by successive adjustments of L_v (by tuning), of L, and of ρ.

4. *Limitations of the Method.*

The most important limitation of the method lies in the fact that the time constant (L/U) of the inductive coil in the arm T cannot readily be made greater than about 0·01. Hence, by (12), putting S=0, the maximum vibration capacity measurable is given by $0{\cdot}01/r$. An examination of the table of vibration constants for Campbell's galvanometers shows that, with the exception of galvanometers H, J_1, J_2, all the vibration capacities are greater than this, so that before the method will apply to these galvanometers the vibration capacities must be reduced.

Since the vibration capacities are inversely proportional to A^2 (*see* equation 5), and A is proportional to the flux density (B) of the air-gap of the galvanometer magnet, the required reduction may be brought about by increasing this flux density. The flux density necessary is given in the last column of the table. It will be seen that these values may be readily obtained by an electromagnet.

5. *Experimental Illustration of Method.*

As a suitable vibration galvanometer was not available, the following arrangement was employed in order to test the method :—

A loop of No. 32 phosphor bronze wire (diameter=0·27 mm.) was stretched between two ivory stops at a distance of 10 cm. apart. Its tension could be adjusted by means of a spring attached to an ivory pulley over which the wire passed. The arrangement was placed between the poles of an electromagnet, the width of the air-gap being about 3 mm.

As source, a small alternator, whose normal frequency was 100 $\sim$ per second, was employed. An oscillogram of the wave-form of this machine showed that it possessed pronounced third and fifth harmonics.

The detector was a Duddell vibration galvanometer. This instrument could not be brought to resonance with the fundamental frequency of the machine, so that in all measurements the galvanometer was tuned to the third harmonic.

The frequency of the fundamental of the source was measured by a reed frequency meter.

Tests were carried out as follows :—

(*a*) With the fundamental frequency of the machine at 100 $\sim$ per second, a current of about $\frac{1}{3}$ ampere was passed along the wire, and its tension adjusted until it was in resonance with the third harmonic. This adjustment could be made with some precision, as the wire emitted a distinct note at resonance.

(*b*) The source frequency was lowered first to 90 $\sim$ per second and then to 80 $\sim$ per second, and the apparent inductance of the wire was measured in each case by Anderson's method.

In this test it was found necessary to bring the third harmonic to resonance and to eliminate the fifth harmonic by means of a Campbell wave-sifter.*

The results obtained were :—

At a frequency of 270 $\sim$ per second, inductance=0·260 mh.
,, ,, 240 $\sim$,, ,, =0·140 mh.

Hence, by (6c) and (7),

Resonating frequency$\fallingdotseq$300, L_v=0·05 mh., C_v=6 millifarads.

* A. Campbell, "Proc." Phys. Soc., Vol. 24, p. 107, 1912.

(*c*) Using the approximate values of L_v and C_v as a guide, the bridge of section 3 was built up as follows :—

The arm T consisted of an Ayrton and Perry variable standard of self-induction of range 4 to 33 millihenries, in series with a condenser of capacity 12·6 mfd. and an adjustable resistance ranging from 0·1 to 10 ohms.

The resistances P and R were fixed at 2·00 and 0·100 ohms respectively. The values of Q and S were obtained by the direct-current adjustments of equations (9) and (10), the fractions of an ohm being obtained by using parallel combinations of resistance boxes. These gave Q=8·75 ohms, S=2·083 ohms. Hence, from (9) and (10),

$$r=0{\cdot}438 \text{ ohm}, \quad U=9{\cdot}55 \text{ ohms}.$$

The alternating-current balance was obtained by successive adjustments of L, the tension of the wire, and ρ; these gave

$$L=30{\cdot}05 \text{ millihenries}, \quad \rho=0{\cdot}90 \text{ ohm}.$$

Hence, from equation (12),

$$r_v=5{\cdot}7 \text{ ohms}, \quad L_v=0{\cdot}0641 \text{ mh.}, \quad C_v=5{\cdot}90 \text{ millifarads}.$$

The resonating frequency of the wire follows from equation (7) as 259 $\sim$ per second. In order to test whether fluctuations of frequency disturbed the balance, the frequency of the source was varied from 70 $\sim$ per second to 90 $\sim$ per second. It was found that, unless the frequency was varied abruptly (thus causing unsteady motion of the wire), the balance remained perfect.

Further, it was found to be quite unnecessary to eliminate either the fundamental or the fifth harmonic of the source.

6. *Values of the Vibration Constants.*

In order to show the magnitude of the vibration constants for actual galvanometers the following table has been prepared from Campbell's data. The resonating frequency is taken in every case as 100 $\sim$ per second. For any other frequency only the vibration inductance will be affected. Its value at any frequency (f) may readily be obtained from the relation $L_v f^2$=constant. Also a variation of the flux density (B) of the air-gap will cause a variation of all the vibration constants, the variation being such that r_v/B^2, L_v/B^2 and C_vB^2 are constant.

Galvanometer A of Campbell's table is omitted. The

vibration resistance of this galvanometer is less than its ohmic resistance, and this condition serves no useful purpose, since, for the most sensitive conditions of the working of a vibration galvanometer, the vibration resistance of the instrument must be at least equal to the ohmic resistance of the working circuit.*

Table of Vibration Constants.

Galvanometer.	r ohms.	r_v ohms.	C_v millifarads.	L_v millihenries.	B	B′.
B_1	6·0	14·1	21·2	0·118	2,500	8,900
B_2	4·6	11·5	16·9	0·148	2,500	7,100
C	10·5	37·5	5·02	0·500	2,500	5,800
D_1	5·7	8·4	43·0	0·058	1,650	8,300
D_2	5·7	25·3	15·6	0·160	2,500	7,500
E_1	6·0	68	3·75	0·667	2,500	3,800
E_2	6·0	94	2·70	0·926	2,500	3,200
F	7·0	168	2·08	1·20	2,500	3,100
G	8·8	342	1·21	2·07	2,500	2,600
H	14·2	2,290	0·251	10·0	2,500	1,500
J_1	14·5	765	0·921	2·72	(1,650)	2,000
J_2	14·5	1,525	0·349	7·16	(2,700)	2,000

r = ohmic resistance of galvanometer.
r_v = vibration resistance.
C_v = vibration capacity.
L_v = vibration inductance.
B = actual flux density.
B′ = flux density required for measurement.

7. *Determination of the Intrinsic Constants.*

By equation (5) a knowledge of r_v, L_v, C_v enables us to determine A^2/β, A^2/γ and α/A^2. In order to obtain α, β, γ, A separately we require another relation. This is supplied by the steady current constant k (defined as the current to produce unit deflection of the coil), which is equivalent to γ/A.

We then have

$$A = L_v k, \quad \alpha = C_v A^2, \quad \beta = S_v A^2, \quad \gamma = kA.$$

Further, the steady current constant will generally give the most rapid method of comparing the flux densities in different air-gaps by means of the relation kB = constant, so that if it is necessary to test a vibration galvanometer suspension in an air-gap other than the one in which it is to be used, this relation, together with the relations in section 6, will be sufficient to determine the vibration constants for any air-gap.

* Butterworth, *loc. cit.*

8. *Conclusion.*

The method given in section 3 has a number of possible uses. The theory, with slight modifications, applies to the motion of any electrically-maintained vibrations—*e.g.*, the motion of the diaphragm of a telephone receiver. Hence the method may be employed to determine the energy losses in such systems. Again, if the galvanometer under test is replaced by a leaky condenser with a series resistance and the condenser in the arm T is removed, the balance obtained in this case will give the values of the leakage, capacity and series resistance.

Finally, the fact that the vibration constants depend on the square of the flux density suggests a method of comparing intense magnetic fields, such as exist in the air-gaps of dynamos, by a null method.

I wish to thank Prof. Gee, Mr. J. Hollingworth, and Mr. A. Campbell for valuable criticism and advice, and Mr. White for the construction of the vibrating system.

ABSTRACT.

The methods usually employed in the determination of the constants of a vibration galvanometer involve the measurement of a deflection under three different conditions. Two of these deflections can only be obtained very approximately.

By extending the theory of the vibration galvanometer it is shown how the constants may be determined by methods which involve only the measurement of one deflection. The remaining measurements are carried out on an alternating-current bridge, and the results obtained are practically independent of the wave-form of the source.

The principle of the method depends on the fact that a vibration galvanometer behaves as a parallel combination of a conductance, a capacity and an inductance, in series with a resistance. It is shown how to balance such a combination, and the method is illustrated experimentally. The constants of various galvanometers are quoted in order to show the applicability of the method. Other uses of the bridge are suggested.

DISCUSSION.

Mr. A. Campbell remarked that it was most interesting to find that the electrical behaviour of a circuit capable of dynamical resonance could be imitated exactly by putting in parallel a resistance, a condenser and an inductance. It was a pity that this combination could not be realised in practice since the inductance must have zero resistance. However, Mr. Butterworth got over the difficulty by his special form of bridge. The limitations of this bridge somewhat lessened the range of application to practical cases, and he hoped that the author would be able to modify the bridge so as to remove these limitations.

Mr. D Owen stated that he had found no difficulty in maintaining the frequency of the source sufficiently steady to maintain the voltage sensitivity of a vibration galvanometer constant within one or two per

cent. for a considerable time. The author's analysis of the vibrating coil was very ingenious, but one would like to know whether sensitivities calculated by his rather complex bridge agreed with those obtained by the usual direct method.

Dr. R. S. Willows asked whether the method could be used to find the dielectric constant of a slightly conducting liquid; if so, was it sensitive enough to be practically useful and did the largeness of the required inductance again limit its applicability.

Mr. Butterworth, in reply, stated that the method would apply to any vibrating system provided that the conditions mentioned in the Paper could be satisfied. It was true, as Mr. Owen had pointed out, that the *voltage* sensitivity of certain galvanometers could be determined very precisely. This held in the case of instruments capable of developing a high back E.M.F. The reduction of current at resonance would then prevent any considerable rise in the vibration in spite of the large increase in *current* sensitivity. The application of the method to the measurement of small capacities required investigation.

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